Lidov-Kozai mechanism in 3:2 and 1:1 resonances

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Introduction

Lidov-Kozai mechanism (LKM) is a secular perturbation which causes long-period oscillations in an orbital inclination ***i*** and eccentricity ***e,*** depending on a perihelion argument ω. LKM was first described by Lidov [1] in the analytical theory for artificial Earth satellites with high orbital inclinations to the ecliptic plane. Later, Kozai [2] developed a similar analytical theory for asteroids. According to these works e maximum and accordingly ***i*** minimum is observed at ω = 90º, 270º. The LKM in regions of resonances was studied by Kozai [3] using a semi-analytical method. Variations of ***e*** and ***i*** as functions of ω were estimated graphically. It was shown that in the resonant regions the LKM manifests itself differently than in non-resonant ones. For 3:2 resonance, i.e. in the Hilda group, it was found that e takes its maximum and ***i*** takes its minimum at ω = 0º, 180º. For 1:1 resonance, i.e. for Trojans, Y. Kozai showed that e maximum is achieved with 2ω = 280º and its minimum with 2ω = 120º.

Secular perturbations

Under the influence of gravitational perturbations from major planets, elements of asteroid orbits change. There are two types of long-period secular perturbations: classical and LKM. These perturbations force eccentricities and inclinations of asteroid orbits to oscillate periodically. In the case of classical secular perturbations, the inclination ***i*** oscillates depending on the longitude of the ascending node **Ω**, and the eccentricity ***e*** - depending on the longitude of perihelion **ϖ.** These oscillations are characterized by forced elements: i_f, Ω_f and e_f, ϖ_f, respectively. As noted above, the LKM ***e*** and ***i*** oscillations depend on the argument of perihelion **ω**. Both types of the secular perturbations act simultaneously. In the region of small inclinations and eccentricities, classical perturbations predominate, but in the region of high inclinations and large eccentricities, the LKM becomes predominant.

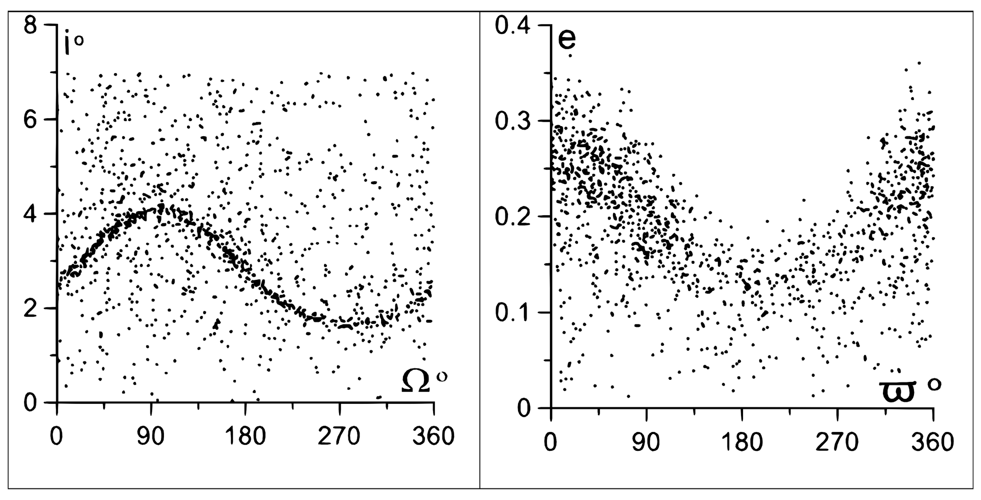

Fig.1 Distributions of osculating elements (i , Ω) and (e, ϖ) for asteroids in the region of the (1911)Schubart family in the Hilda group.

In Fig.1 the distributions of osculating elements for asteroids in the region of the (1911)Schubart family clearly show classical secular perturbations. The dense waves are formed by members of the family. A long-term orbital evolution for every asteroid belonging to the family is in accordance with these curves.

Exclusion of classical secular perturbations

It is possible to separate classical and LKM perturbations in the motion of asteroids, because they depend on different angular variables. To reveal the LKM impact on asteroid orbits, classical secular perturbations should be eliminated from osculating elements. It can be done if the corresponding forced elements are known. The empirical method of proper elements calculation was proposed by Vinogradova [4,5]. This method uses observable distributions of osculating orbital elements to calculate forced ones. It can be applied to both non-resonant and resonant cases if there are sufficient number of asteroids in the region under consideration. Using this method forced elements were calculated for Hildas and Jupiter Trojans. An exclusion of classical perturbations can be made very easy using a coordinate transformation formula, as it was described by Vinogradova [4,5]. The resulting plots of inclinations and eccentricities versus a perihelion argument can show the LKM element's oscillations.

LKM i-oscillations in the region of (1911)Schubart family in Hildas

The forced elements derived by the empirical method for Hildas are as follows: i_f = 1.20º, Ω_f = 99º, e_f = 0.069, ϖ_f =20º. Using these forced elements, proper elements i_p, Ω_p, e_p, ϖ_p, and ω_p were calculated for every asteroid in the region of the (1911)Schubart family. So, proper elements were derived for the whole population at a fixed point in time corresponding to an epoch of osculating elements. Thereafter, the element distributions (i_p, ω_p) and (e_p, ω_p) were built. The LKM ***i*** **-** oscillations are visible rather well in the plot (i_p, ω_p) in Fig.2, but positions of the ***i*** **-** minimums and maximums are shifted by 90º. The corresponding LKM ***e***-oscillation in the distributions (e_p, ω_p) could not be detected by this method because of a negligible oscillation amplitude.

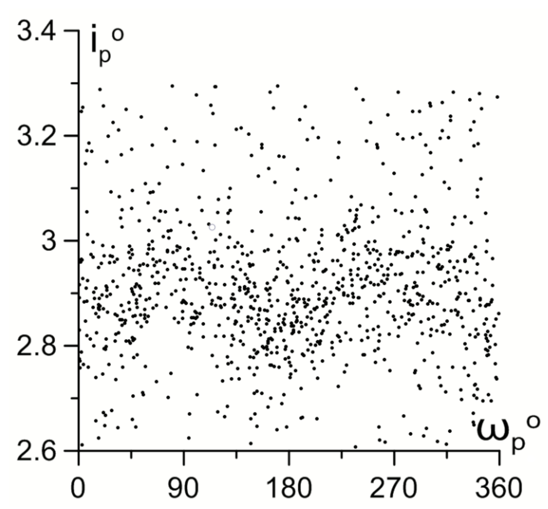

Fig 2. *Lidov-Kozai i-oscillations in the region of (1911)Schubart family in Hildas.*

LKM i- and e-oscillations in the Hildas

The exclusion of classical perturbations can be performed for individual asteroids in the process of numerical integration of asteroid's orbit. As an example, the motion of the asteroid (153) Hilda was considered. To exclude classical secular perturbation, following forced elements were used: i_f ,= 1.3º, Ω_f = 100.5º, e_f = 0.068, ϖ_f = 14º. Here, i_f, Ω_f represent the plane of the Jupiter's orbit, e_f, ϖ_f were calculated specifically for the asteroid (153)Hilda. These forced components were excluded from osculating elements in the process of numerical integration at every print step. The derived values of i_p and e_p were plotted versus ω_p. In Fig. 3 one can see how these elements change under the influence of the LKM. A complete picture of the orbital element variation can be obtained only after several revolutions of ω. A corresponding time interval should include at least one period of the **Ω** revolution. Thereafter, the entire range of possible values of the elements becomes outlined on the plot. The resulting plots show well the LKM element oscillations, and positions of maximums and minimums of the elements can be easily found: e takes its maximum and i takes its minimum at ω = 0º, 180º.

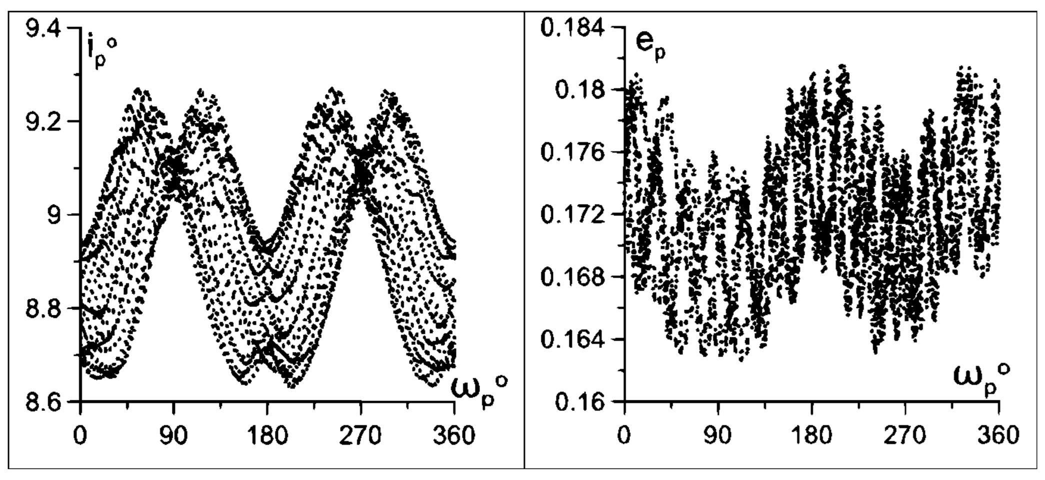

Fig.3. *LKM oscillations of the orbital elements of (153) Hilda derived using* numerical integration.

LKM in the Jupiter Trojans

For Jupiter Trojans the corresponding orbital elements of Jupiter (i_f =1º.3, Ω_f = 100º.5, e_f=0.049 can be taken as forced ones. Forced perihelion longitudes ϖ_f for two populations of the Trojans are different. It was shown by Vinogradova [4], that for L4 Trojans $\varpi_f = \varpi_{jup}$ + 60º = 74º, whereas for L5 Trojans $\varpi_f = \varpi_{jup}$ - 60º = 314º (ϖ_{jup} = 14º.0). After eliminating the classical secular perturbations, the plot of i_p and e_p against ω_p can show the LKM oscillations. Such plots in Fig. 4 show well how the orbital elements of the asteroid (15527) 1999YY2 from the L4 region and (1873) Agenor from the L5 region change under the action of the LKM. The orbits of these asteroids are characterized by rather large eccentricities and inclinations. Therefore, amplitudes of the element oscillations are significant and well visible in the plots. Positions of the e maximum (or i minimum) for Trojans were found to be different from that in both non-resonant regions and Hildas. Moreover, their positions are different for two groups of Trojans. For L4, the e maximum is shifted by - 60º relative to the normal position and is located at the points ω = 30º , 210º . For L5, it is shifted by + 60º and is located at ω = 150º , 330º .

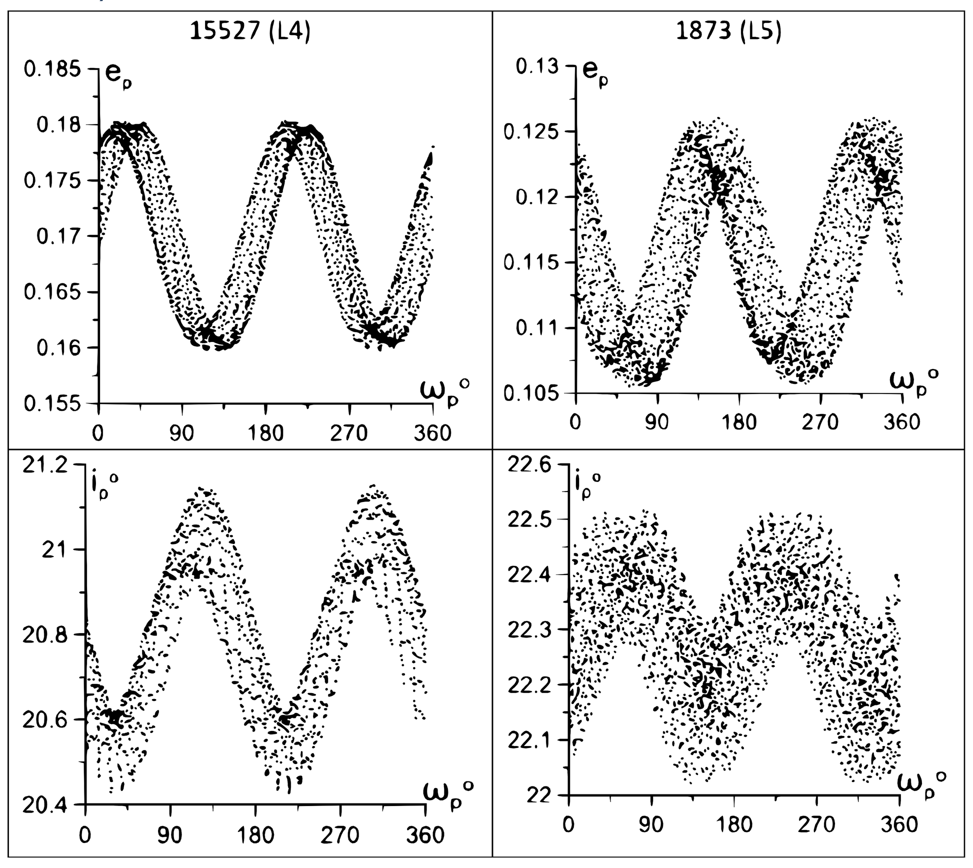

Fig 4. *Lidov-Kozai oscillations of the orbital elements of (15527) 1999 YY2 in the L4 region (left panels) and (1873) Agenor in the L5 region (right panels).*

Conclusion

It was found that the LKM effect in these resonant regions differs from that in non-resonant ones. For the vast majority of the Hildas, the e maximum and accordingly the i minimum is observed not at the usual values of the perihelion argument (ω = 90º, 270º), but about ω = 0º, 180º. Thus, the usual plots are shifted by 90º. For Jupiter Trojans the usual plots are also shifted, but by ± 60º.The e maximum is achieved with ω = 30º, 210º for L4 Trojans, and with ω = 150º, 330º for L5 Trojans.

These findings were compared with Kozai [3] results computed by a semi-analytical method. The results, obtained for the Hilda group by different methods, matched. Regarding the Jupiter Trojans, the result derived for L5 Trojans agrees partially with those by Y. Kozai: it is the same for e minimum and different for e maximum. Whereas, the result concerning the L4 Trojans was not obtained in Y. Kozai's work.

Литература

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