K.V. Kholshevnikov and the Euler-Lambert problem of constructing the orbit of a body based on its two positions

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Abstract. In the report, the author first shares personal memories of Konstantin Vladislavovich Kholshevnikov as a person. Then the author describes a new method for solving the Euler-Lambert problem, which is one of the main problems of celestial mechanics and to which K.V.Kholshevnikov paid attention in one of his works.

In the first part of the presentation, first of all, the author expresses his deep respect to Professor K.V. Kholshevnikov. The author was lucky enough to meet with K.V. repeatedly, mainly at scientific events. K.V. was a wonderful person - with a constant smile, friendly to colleagues, the "soul" of the team, always created a warm, friendly "aura" around himself. And at the same time, he had encyclopedic knowledges, was a worthy scientist of high, international class. When analyzing complex problems of celestial mechanics, he was able to combine the construction of a clear mathematical statement of the problem and a rigorous mathematical approach to its solution, the ability to find simple methods for solving the problem.

• The second part of the presentation notes the analysis made by K.V. for two important celestial-mechanical problems. This is, firstly, an analysis of the "modern" mitigation problem of ensuring the asteroid-comet safety for the Earth. This analysis was presented by K.V. at a scientific conference together with T.N. Sannikova and a scientist of the M.V. Keldysh IAM Professor V.M. Chechetkin. In the presence of a controlled space influence on a dangerous celestial body, the equations of motion for the body become complex, difficult to analyze them. The authors average these equations, simplify them, and then analyze the multi-revolutions motion of a dangerous body, obtaining a number of results important for practice.

• The second problem is a classic problem that is stated in the 18th century by the great L. Euler. This is the task of determining the orbit of a celestial body by its two positions $\mathbf{r_1}, \mathbf{r_2}$ at given times t_1, t_2. A separate paragraph of chapter 4

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"Determining orbits" of the book "The Problem of two Bodies" [1], published in 2007 as a textbook by K.V. Kholshevnikov together with a colleague V.B. Titov and presented to me in October 2017. The solution to this classic problem given in this book is interesting in two ways. First, the authors, following to I. Kepler and I.K.F. Gauss, are shown the "physics" of the solution, and then given a mathematical algorithm of the solution of the problem. At the end of this iterative algorithm, the authors analytically show (this is very rarely doing) that the solution exists and is the only one (for the considered special case, at a flight angle of $0 < \varphi < \pi$. The specified Euler-Lambert problem is important for astronomy and classical celestial mechanics for preliminary (without taking into account perturbations) determination of the orbits of natural celestial bodies (asteroids, comets), as well as for astronautics, space flight mechanics – for preliminary design construction of the orbit of a spacecraft during flight on a time interval (t_1, t_2) from the orbit of one celestial body to orbit another celestial body [2-4].

• Due to the importance of this problem, many methods have been developed to solve it. A comparative analysis of some methods is given, for example, in the works of M.F.Subbotin, P. Escobal [5-6]. Most of the developed methods for solving the problem are usually based on the fact that the set of flight orbits between two points in the central field forms a one-parameter family of orbits with flight between specified points in some time $(t'_2 - t_1)$. Depending on the choice of the parameter of this family, we obtain different methods for solving this Euler-Lambert problem. V.A. Egorov, apparently, for the first time drew attention to the fact that the results of D.E. Okhotsimsky on the analysis of ballistic flights can be used to construct a simple and very visual, new method for solving the Euler-Lambert problem. Therefore, this method is often called the Okhotsimsky D.E.-Egorov V.A. method. In the Okhotsimsky – Egorov method, the angle of inclination θ_1 of the initial velocity to the initial transversal is taken as a parameter of the specified family [2-4]. Knowing the angle of flight in the plane of the orbit, as well as the initial and final distances to the center of gravity $\mathbf{r_1}, \mathbf{r_2}$, allows us to determine the value of the initial flight velocity of the body $V1$ and all the parameters of the orbit, including the flight time Δt. Iteratively, we select the initial angle of inclination of the speed so that the flight time is equal to the specified time. Analysis has shown that this method has good convergence characteristics and solutions to the Euler-Lambert problem.

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