«AMCM» Conference Didicated tu
Memory of Professor K.V. Kholshevnikov
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K.V. KHOLSHEVNIKOV and THE EULER-LAMBERT PROBLEM OF CONSTRUCTING THE ORBIT OF A BODY BASED ON ITS TWO POSITIONS

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1. Happy Memory on Konstantin Vladislavovich

2. The Mitigation Problem of Ensuring the Asteroid-Comet Safety of the Earth

[Sannikova T.N., Kholshevnikov K.V., Chechetkin V.M. Using the Gauss Averaging Method for Analysis of a Celestial Body Deflection by a Low Thrust // ISSN 621.396. Ecological Bulletin of Research Centers of the BSEC. 2013, N4, том 2, сс. 144-147]

3. The Euler-Lambert Problem of determining the orbit of a celestial body by its two positions

[K.V. Kholshevnikov, V.B. Titov. The Problem of two Bodies. St. Petersburg, State University. A Text-book. 2007. 180 p.]. Chapter 4 "Determining orbits“. P. 4.3. Determining the Orbit by the Positions. 4.3.2. Determining the orbit by two positions [1].

4. Ideas of the Method – 1 –

Firstly, it was L. Euler who stated the Euler-Lambert problem and solved it for one case in 1743. I. Lambert in 1761 – 1771 гг. developed it. The E.-L. problem is important for Astronomy and Astronautics. It was worked out a lot the Methods to solve it ([1-6], et al.).

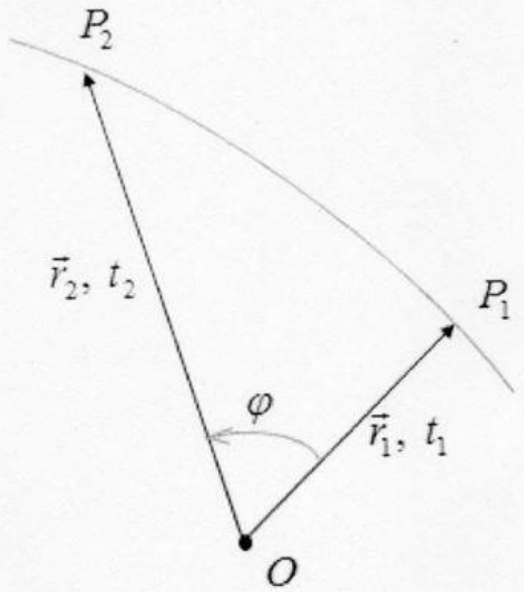

Рис.1

Fig. 1 gives a flight between Points P_1, P_2.

4. Ideas of «ballistic» Method to solve the E-L problem -2-

It is clear Method if the parameter of orbit family for flight between two given points is an angle θ_1 of inclination of initial velocity V_1 to the initial transversal. Fixing this angle θ_1 gives this velocity скорость V_1 by the D.E. Okho-tsimsky formula:

$$2\beta_1 = \frac{1 - cos\varphi}{\left(\frac{r_1}{r_2} - cos\varphi\right) cos^2\theta_1 + sin\varphi \cdot sin\theta_1 cos\theta_1}, \quad (1)$$

here value $\boldsymbol{\beta_1}$ is ratio of initial velocity squared and initial parabolic velocity squared.

4. Ideas of Okhotsimsky-Egorov Method to solve the Euler-Lambert problem – 3 -

The Inclination of initial velocity to transversal and the Value of initial velocity determine completely the orbit for flight between two given points. Varying the angle θ_1, we find its value, for which the flight time is equal to the given time, this solves the problem. This idea of solving the problem firstly, obviosly, has given V.A. Egorov. That is why, we suggest to call this method of solving the Euler-Lambert problem by the Okhotsimsky-Egorov Method.

4. Ideas of Okhotsimsky-Egorov Method to solve the Euler-Lambert problem – 4 -

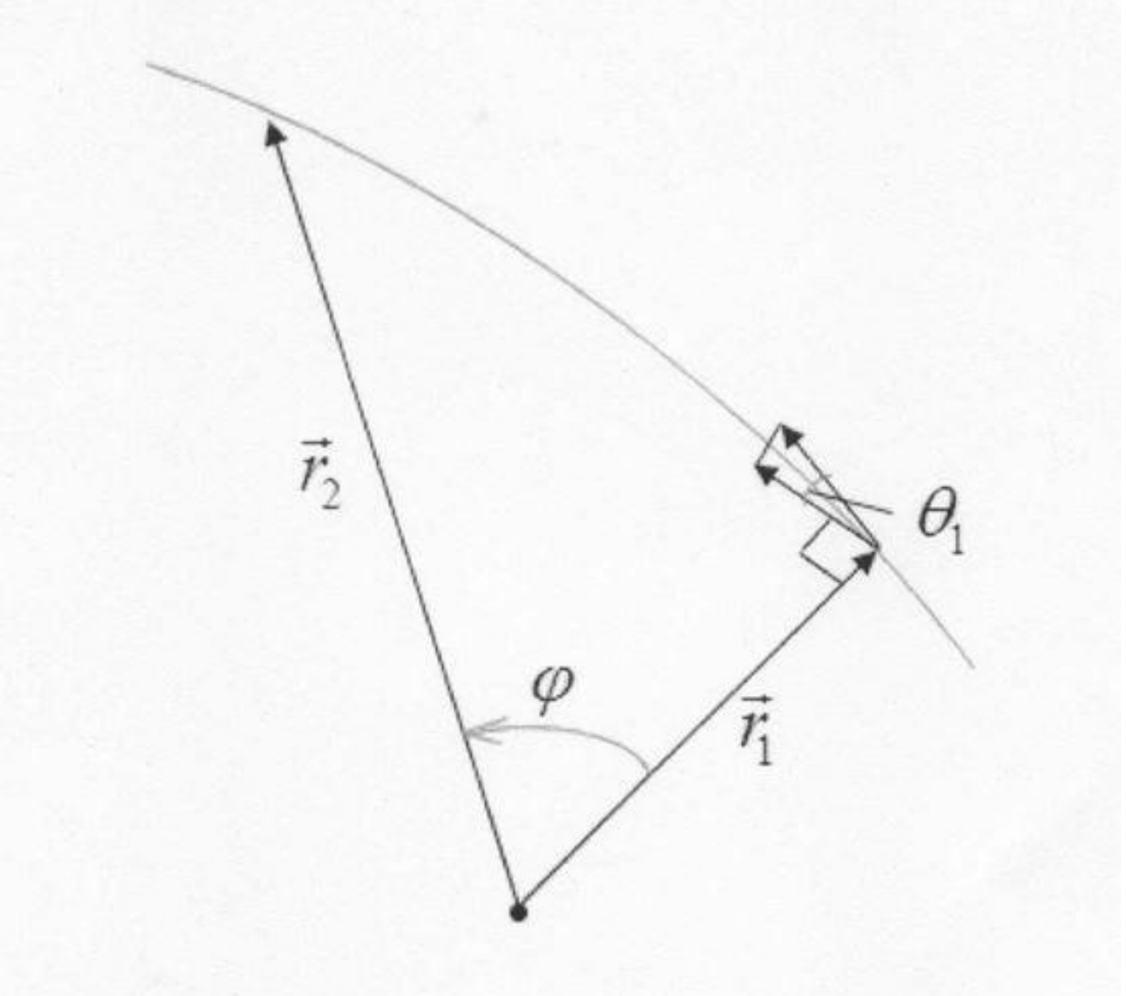

Figure 2

5. Basics of the Okhotsimsky-Egorov method algorithm-1-

Some characteristics of algorithm:

1. **Two value of flight angle and two orbites are defined**
2. **A parameter of flight orbit family is an angle θ_1 of inclination of initial velocity to initial transversal. This angle determines the velocity value by formula.**
3. **This angle and flight orbit for given flight time t_2- t_1 are determined by iterations. For initial step, for this angle we take, for example, an angle value θ_{10} for minimal initial velocity.**
4. **For the flight at a single revolution, solving the problem exists and the solution is one, the only.**

5. Basics of the Okhotsimsky-Egorov method algorithm-2-

- **Chord s =** $P_1 P_2$

$$s = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\varphi} \qquad (2)$$

- **Minimal value of semi-major axis** a_0

  - $a_0 = (r_1 + r_2 + s)/4$; **(3)**

- **Minimal constant of energy and minimal initial velocity**

  - $h_0 = -\mu/a_0$; $V_{10}^2 = \frac{2\mu}{r_1} + h_0$ **(4)**

5. Basics of the Okhotsimsky-Egorov method algorithm -3-

- **Minimal initial Velocity parameter:**
-
$$\beta_{10} = \frac{V_{10}^2}{2\mu/r_1} \qquad (5)$$

- **«Optimal» angle for minimal initial velocity:**
- $$tg2\theta_{10} = \frac{r_2 \sin\varphi}{r_1 - r_2 \cos\varphi};\ tg\theta_{10} = \frac{\beta_{10} \sin\varphi}{1-\cos\varphi} \to \theta_{10} \qquad (6)$$
- **Fixing (in the cycle, from the family) an angle of the inclination of initial velocity to initial transversal**
- $\theta_1 = \theta_{10} + \Delta\theta,\ \Delta\theta_{min} \le \Delta\theta \le \Delta\theta_{max}$ **…………………. (7)**
- **The initial velocity, by D.E. Okhotsimsky formula (1):**

$$2\beta_1 = \frac{1 - cos\varphi}{\left(\frac{r_1}{r_2} - cos\varphi\right) cos^2\theta_1 + sin\varphi \cdot sin\theta_1 cos\theta_1},$$

5. Basics of the Okhotsimsky-Egorov method algorithm -4-

- **Varying the angle θ_1 of inclination of initial velocity (by I. Newton, for example), we find its value, for which the flight time is equal to the given time $\Delta t_{\text{дан}}$:**
- $$\Delta \boldsymbol{t}(\boldsymbol{\theta_1}) = \Delta \boldsymbol{t}_{\mathbf{giv}} \,. \qquad (8)$$
- **This solves the problem.**

6. Conclusions

1. **D.E. Okhotsimsky Results on the dynamics of ballistic motion can be used, , in accordance with the ideas of Egorov V.A., as the basis for the method for solving the Euler-Lambert problem.**
2. **We suggest to call this method of solving the Euler-Lambert problem by the Okhotsimsky-Egorov Method.**
3. **The Okhotsimsky-Egorov Method attracts our attention by its clarity and good convergence of iterations.**

5. Список литературы -1-

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