Lerman separatrix map for the problem of satellite attitude motion

Vladislav Sidorenko

Abstract. The attitude motion of an axisymmetric satellite under the influence of a gravitational torque is studied. The satellite's center of mass moves in a circular orbit in a central gravitational field. If the projection of the satellite's angular momentum vector onto its axis of symmetry is zero, then so-called "planar" motions are possible. In planar motions the axis of symmetry moves in the orbital plane and, therefore, the angular velocity vector of the satellite is perpendicular to this plane. To analyze the properties of the motions of the satellite, which are close to planar ones, perturbation theory is applied. A map is constructed that approximates the map generated by the phase flow of the system. Using this map, we were able to establish some previously unknown properties of the satellite's attitude motion in a gravitational field.

1. Problem formulation

The investigations of the attitude motion of natural and artificial celestial bodies is an important area of space flight mechanics and celestial mechanics [1].

The aim of our study is to analyze the properties of the motion of an axisymmetric satellite relative to its center of mass under the influence of a gravitational torque. It is assumed that the satellite's center of mass moves in a circular orbit in a central gravitational field.

Let **L** be the vector of the angular momentum of the satellite relative to its center of mass O. If the projection of **L** onto the symmetry axis of the satellite is zero, then so-called "planar" motions are possible - motions in which the symmetry axis is always in the orbital plane, and the angular velocity vector is perpendicular to this plane. In the phase space of a Hamiltonian system with two degrees of freedom, which describes the motion of an axisymmetric satellite relative to the center of mass, planar motions are associated with phase trajectories lying on a two-dimensional invariant manifold. The behavior of phase trajectories on this manifold is similar to the behavior of trajectories on the phase portrait of a mathematical pendulum - separatrices separate trajectories corresponding to the rotations and oscillations of the satellite relative to the local vertical.

In [2, 3], the stability of planar motions of an axisymmetric satellite was studied. We tried to describe in as much detail as possible the dynamics of the system in the case when the phase trajectories are located in the vicinity of the separatrix contour.

2. Methods

L.M. Lerman developed a general approach to study Hamiltonian systems with two degrees of freedom, in the phase space of which there is an invariant manifold with separatrix loop [4]. The main idea of this approach is to construct, using perturbation theory methods, a map that approximates the map generated by the phase flow of the system.

The approximating map is obtained as a composition of a rotation operator that describes the behavior of the phase flow in the vicinity of an unstable equilibrium (which is part of the separatrix loop), and a linear map that describes the behavior of the phase flow in the vicinity of the separatrices. The map depend on parameters, finding the values of which is a separate task.

Lerman's approach was used in [5] to analyze the dynamics of a specific mechanical system - certain version of a double pendulum. Planar oscillations of an axisymmetric satellite near the local vertical differ from oscillations of a pendulum by the physical nonequivalence of situations corresponding to different directions of relative angular velocity. Therefore, we needed to introduce a number of modifications to the construction of the separatrix mapping used in [4, 5].

Another approach to constructing a map that approximates the phase flow for the similar problem can be found in [6, 7]. It should be noted that in [6, 7] the center of mass of the satellite moves in an elliptical orbit, and the consideration is limited to planar motions only.

3. Results of investigations

A map was constructed that approximates the phase flow in the problem of attitude motion of axisymmetric satellite. Its correctness was checked by comparison with the numerically constructed Poincaré sections of the phase flow.

Using this map, we were able to describe a series of bifurcations, as a result of which families of spatial periodic motions of the satellite are born from planar motions. The stability of the found families of periodic motions is studied for different values of the ratio of the longitudinal and transverse moments of inertia of the satellite.

The stability of the separatrix loop separating the planar rotational and oscillatory motions of the satellite has been studied. The critical value of the ratio of the moments of inertia of the satellite is found, at which the loss of stability of this separatrix loop occurs.

Also the fractal nature of the dynamical structure of the phase space of the problem was revealed (by dynamical structure we mean, in particular, stationary and periodic solutions, stable and unstable invariant manifolds adjacent to these solutions).

All this allows us to conclude that the constructed map makes it possible to carry out a detailed study of the dynamics of an axisymmetric satellite in a significantly more efficient manner in comparison with previously used approaches.

References

[1] V.V. Beletsky, *Motion of an artificial satellites around its center of mass.* NASA, 1966.

[2] A.P. Markeev, *Stability of planar oscillations and rotations of the satellite in circular orbit.* Cosmic Research. 1975. V. 13. P. 322–336.

[3] A.P. Markeev, B.S. Bardin, *On the stability of planar oscillations and rotations of a satellite in a circular orbit.* Celestial Mechanics and Dynamical Astronomy. 2003. V. 85. P. 51–66.

[4] L.M. Lerman, *Hamiltonian systems with loops of a separatrix of a saddle-center.* Sel. Math. Sov. 1991. V. 10. P. 297–306.

[5] C. Grotta Ragazzo, *On the stability of double homoclinic loops.* Commun. Math. Phys. 1997. V. 184. P. 251–272.

[6] A.A. Burov, *On the oscillations of a satellite in an elliptical orbit.* Cosmic Research. 1984. V. 22. P. 133–134.

[7] I.I. Shevchenko, *The separatrix algorithmic map: application to the spin-orbit motion.* Celestial Mechanics and Dynamical Astronomy. 1999. V. 73. P. 259–268.

Vladislav Sidorenko
Keldysh Institute of Applied Mathematics RAS
Moscow, Russia
e-mail: `sidorenk@keldysh.ru`