Towards understanding the astronomical orientation of the Old Kingdom pyramids

Irina Tupikova

Lohrmann Observatory, TU Dresden & Max Planck Institute for the History of Science, Berlin.

Abstract. The northern orientation of some of the Old Kingdom pyramids was formerly presumed to be a result of the pyramids having been aligned to the position of the North Celestial Pole. In 1984 an important observation was published by S. Haack that the orientation of the pyramids of the Fourth Dynasty follows a special pattern of digression from true north and this progressive deviation from true north direction was understood to be a consequence of the pyramids having been aligned to a star whose celestial position changed due to the effect of the general precession of the rotational axis of the Earth. Instead of a single star, later publications considered a possible usage of some notable stellar configurations exhibiting a similar azimuthal trend; these recent attempts at explanation forced researches to shift the conventional Old Kingdom chronologies by some significant (and different) number of years. We propose and discuss two new solutions whose systematic errors are minimal among all the known proposals: one based upon the vertical alignment of Kochab and ζ UMi and another one upon the horizontal alignment of Alioth and Mizar. In contrast to other methods, the former pair has the advantage that it could have been observed at lower altitudes. Both variants show an impressive degree of agreement with the trend in the orientation of the pyramids for von Beckerath's (lower estimates) chronology and do not demand any temporal shift in dating of the pyramids.

1 Pattern in the orientation of the pyramids

The existing pyramids of the 4th Dynasty (Snofru's, Khufu's, Djedefre's, Khafre's, and Menkaure's) as well as three pyramids of the 5th Dynasty (Sahure's, Neferirkare's, and Unas') seem to be prime examples of using the modern cardinal directions and the measured precision of their orientation apparently contradicts the common modern consensus that the ancient observations were more qualitative than quantitative. The precision of their orientation towards the cardinal directions (for available measurements, in arcminutes) is given in cols. 3–6 of Table 1 where the deviation dA from the corresponding cardinality (i.e. from true east by the north- and south sides and from true north by the west- and east sides) is counted as positive in the clockwise direction and negative otherwise.[1]

[1] The data for the orientation of the pyramids can be found in the following sources: Djoser's pyramid in Lauer (1960, 99) and Romer (2007, 279), Snofru's Meidum in Petrie (1892, 6), Snofru's Bent in Dorner (1986, 51), Snofru's Red in Dorner (1998, 30), Khufu's in Nell and Ruggles (2014, Table 1b), Djedefre's in Mathieu (2001, 458), Khafre's in Nell and Ruggles (2014, 322, Table 3b), Menkaure's in Nell and Ruggles (2014, Table 5c, the numbers are mean values for different courses), Sahure's in Arnold (1991), Neferirkare's in Žába (1953, 1), and Unas' pyramid in Dorner (1981). It is well known that different casing and masonry levels may show different orientation (Petrie 1883, 37).

Ruler	*Dynasty*	*dA (north side)*	*dA (south side)*	*dA (west side)*	*dA (east side)*
Snofru-Meidum	4	-35.4' ± 0.4'	-23.6' ± 0.4'	-18.1' ± 0.4'	-20.6' ± 0.4'
Snofru-Bent	4	-7.5' ± 0.2'	-4.2' ± 0.2'	-11.8' ± 0.2'	-17.3' ± 0.2'
Snofru-Red	4				-8.7' ± 0.2'
Khufu	4	-3.6' ± 0.3'	-0.5' ± 0.3'	-3.7' ± 0.3'	-3.4' ± 0.3'
Djedefre	4	-51.7' ±1'	-48.4' ± 1'	-50.8' ±1'	-43.9' ± 1'
Khafre	4	-3.8' ± 0.3'	-5.8 ± 0.3'	-4.2' ± 0.3'	-4.0' ± 0.3'
Menkaure	4	20'	32'	25'	12'
Sahure	5				-23' ± 10'
Neferirkare	5				30' ± 10'
Unas	5			17.4' ± 0.2'	17.1' ± 0.2'

Table 1: Orientation of pyramids: deviation from cardinal directions.

Obviously, if the orientation *intentionally* matched the north-south or east-west direction, the method used for the orientation of pyramids must have been an astronomical method, because the concept of such cardinality *per se* is an astronomical concept. In fact, the orientation of the old Kingdom pyramids does not tell us whether the intended target was the conceptual "Pole" or a star/stellar configuration fortuitously sighted in that direction; the latter orientation could be mistaken for our modern concept of cardinality.

In 1984, S. Haack noticed that the orientation of the pyramids of the Fourth Dynasty follows a special pattern of digression from true north. This trend is a very useful guideline because it permits refutation of the astronomical methods which do not show a secular trend over time. We can exclude, for example, the North Celestial Pole (NCP) as a possible target of observation—the position of the Pole is fixed for a given position on Earth and changes only due to geophysical factors. Some authors (e.g. Herschel in 1887) have argued that Thuban (α Draconis, apparent magnitude 3.67) was used to define the north direction, but such an assumption attributes to this relatively faint star a very important role not confirmed by textual evidence and even the identification of this star in the Egyptian texts is questionable. More than that, in the range of the widely agreed upon chronologies the maximal azimuthal deviation of Thuban from true north varied between $\pm 1°$ and $\pm 1° 40'$ and couldn't support the precision of the orientation of the pyramids. We can also exclude solar observations: methods based upon observing the direction of the Sun at the moments of its equal elevation above the horizon (e.g. at rising and setting) followed by halving of the observed arcs or angles, do not show a secular trend. The Sun could also not have been observed at culmination, because at that moment the Sun crosses the meridian of the location, that is, always marks the true north-south direction. Similarly, the alignment procedure could not have been based upon the observation of stars at equal elevations or at their upper or lower culminations.

Within a relevant range of accuracy, two effects (besides the daily rotation around the NCP) change the apparent stellar positions—the proper motion of stars, and the general precession of the rotational axis of the Earth. The apparent effect of precession for an observer at a given geographical location depends upon the right ascension and declination of stars; it manifests itself for every particular star and for every stellar configuration differently. Due to the effect of precession, stellar azimuths are time-dependent and can only be calculated for specified calendar dates. Several chronologies of the period are available; in this text we have used the three most agreed-upon chronologies—von Beckerath's (1997),[2] Malek's (as given in Shaw 2000) and Hornung et al. (2006), all modified according to Stadelmann's (1986) proposal by having 48-years for the duration of Snofru's reign. The corresponding data in years BC are given in Table 2 where the dates in square brackets for the construction of Snofru's Bent and Red pyramids follow the temporal proportions given in Stadelmann (1986) which were also used by Spence (2000).

[2] We use here the lower range of dates given by von Beckerath (1997).

Ruler/pyramid	*Dynasty*	*Acession date (Shaw)*	*Acession date (von Beckerath)*	*Acession date (Hornung et al)*
Snofru-Meidum	4	2637	2602	2558
Snofru-Bent	4	[2620]	[2583]	[2541]
Snofru-Red	4	[2609]	[2572]	[2530]
Khufu	4	2589	2554	2509
Djedefre	4	2566	2530	2482
Khafre	4	2558	2522	2472
Menkaure	4	2532	2489	2447
Sahure	5	2487	2446	2428
Neferirkare	5	2475	2433	2415
Unas	5	2375	2317	2321

Table 2: Accession dates according to Shaw's, von Beckerath's, and Hornung's chronologies with 48 years for the reign of Snofru.

The special pattern of digression of the pyramids of the Fourth Dynasty from true north is displayed in Fig. 1 where the y-axis gives the azimuths[3] of the east sides of the pyramids and the time-axis follows von Beckerath's chronology; the trend, of course, exists also for other chronologies although the gradients of the lines become slightly different. What S. Haack identified as a temporal correlation of the orientation of the pyramids was the trendline 'c' (the 'main' trendline) running as a guide to the eye through the azimuths accurately surveyed at that time Snofru's, Khufu's, Khafre's, and Menkaure's pyramids; the azimuths of Djedefre's, Sahure's, and Unas' pyramids lie along the line 'd' as shown in Fig. 1.[4] The presence of two trendlines in the orientation of the pyramids is an important aspect of the problem. No doubt that the procedure was essentially a religious matter, carried out according to strict rituals at fixed dates, but the pattern of orientation of the pyramids displayed in Fig. 1 makes it also evident that either the same stellar configuration (or a single star) was the target at different moments of time or it was not always the same target object which was used in the alignment of the pyramids.

[3] In this graph and further in the text we use the mean values of the azimuths of the pyramids' sides given in Table 1 with the confidence intervals kept in mind. Following K. Spence (2000, 320), it is assumed here and further in the text that "the pyramid alignment ceremony occurred in year 2 of each king's reign (with the exception of those for the later pyramids of Snofru)".

[4] S. Grigoriev (2015, 2) was possibly the first to point towards the existence of the trendline in the orientation of these pyramids. Due to a poor state of preservation, the data on their orientation are much less accurate than those of the three pyramids in Giza and will be shown on the graphs only for the aim of comparison.

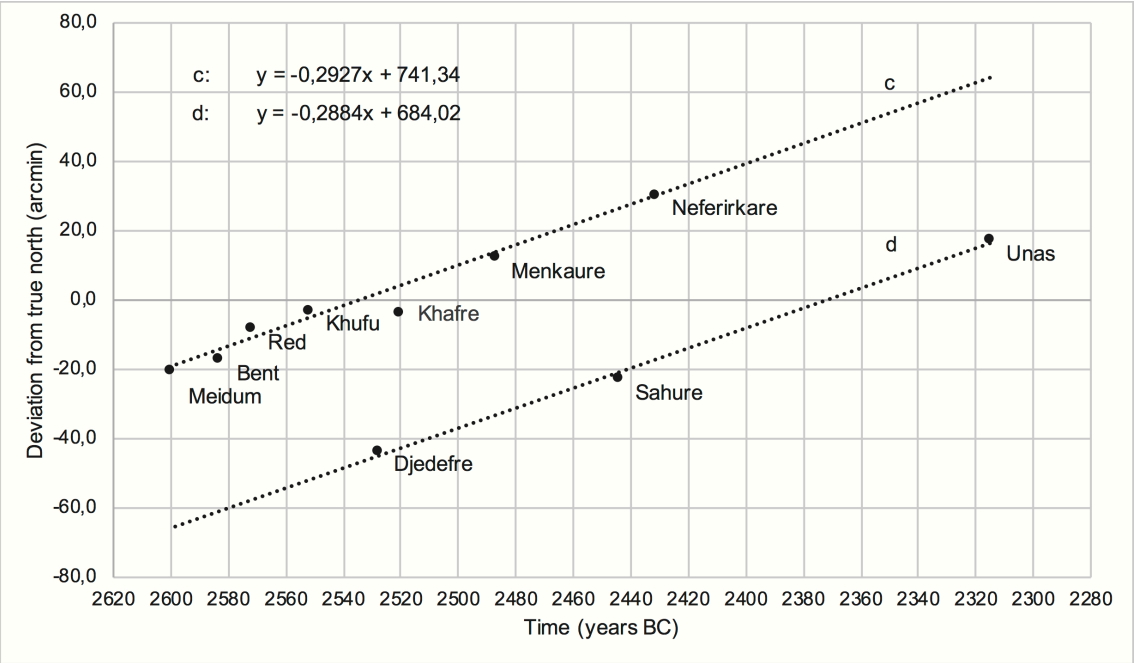

Fig. 1: Orientation of pyramids: deviation from true north over time.

Assuming that over the time of the Old Kingdom, general precession was responsible for the secular trend in the alignment of the pyramids, S. Haack proposed that the primary alignment direction was true east, which was determined by observation of β Scorpii as first visible at its rising. Such an explanation presumes, however, that this cardinal direction was already known: otherwise, one cannot explain why the orientation of the pyramids was based upon adjustment to a relatively faint, not very prominent star in the east. Because there was no especially bright star rising close to the true east direction, we can argue that neither the northern nor the southern sides of the pyramids were oriented along the east-west direction but rather the eastern or western sides, or both of them, were oriented with the help of stellar observations.

No documents have yet been found that can help us to understand exactly how the architects of the Old Kingdom oriented any of their constructions. Some later texts of the Helleno–Roman period report that foundations of the temples were established during a ceremony called the *Stretching of the Cord* when a primary direction was laid out by stretching a rope between two stakes or poles. The earliest mention of this ceremony refers to an unknown king of the 1th Dynasty and is presented in the basalt slab known as the Palermo Stone (which records two similar events)—thus, the ceremony may be even older than the pyramids (Belmonte 2001, S7).

An important inference can be drawn from the descriptions of the *Stretching of the Cord* ceremonies: 1) that the asterism *Meskhetiu*[5] played an important role in this ceremony, 2) that the *merkhet* (a bar with a plumb line attached to a wooden or bone handle) was somehow used in the procedure and that the goddess of time was involved (possibly, to mark a specific moment of observations). The related texts tell us about marking all four corners of the construction in course of a single ceremony; thus, we can assume that either only one side was oriented astronomically and the others have been aligned to it by geometrical means, or two sides were oriented simultaneously by two surveyors, or that two different configurations of stars were used to align the west- and the east sides of the pyramids independently (of course, the

[5] This culturally very important constellation is nowadays commonly agreed upon to be the asterism of the Big Dipper in the constellation of the Ursa Major.

possibility that the centre line of foundations was oriented first cannot be excluded). Whereas the direction determined astronomically contains only the errors of astronomical observations, the orientation of the sides of the pyramids adjusted to this direction by geometrical means depends also upon unknown errors in realization of parallel constructions and right angles. In the following discussion, we will consider the accuracy of the orientation of the east sides (this set of data is also better known); the orientation of the west sides will be discussed later in the text.

2 Targeting stellar configurations

Of course, there exist many stellar positions which can explain the orientation of any single pyramid. But to propose a candidate for a single pyramid would not explain the trend in the orientation of the pyramids: it is the main trendline of their azimuths which serves as a constraint to the possible solutions. Clearly, the position of a star or a stellar configuration used as a target for orientation of pyramids should have been specified by some rule: it could not have been observed at an arbitrary moment of time because in this case, the orientation of pyramids would not show any trend. This unavoidably leads us to conclude that such stars must have been observed at a particular and remarkable stellar alignment. The simplest noticeable geometrical forms built up by stars are their linear alignments, e.g. vertical and horizontal arrangements. Astronomical observations of both variants could be realized with simple instruments following natural physical laws: verticals with plumb lines, and horizontal lines with the help of water levelling.

The method of orientation based upon a vertical alignment of stars proposed by Kate Spence in 2000 was that "the pole was considered to be located on an invisible chord linking two circumpolar stars on opposite sides of the pole. These two stars rotate around the pole, and when they are vertically aligned above the north horizon (one at its upper culmination and another at its lower) an alignment made towards these stars with a plumb line will be exactly oriented to true north, as long as the chord itself passes precisely through the pole"(Spence 2000, 322). The author apparently presumed that the concept of the NCP must have existed at the time of the construction of the pyramids and that their sides were oriented according to this epistemological concept—but the concept of the Celestial Pole is not known to have existed in ancient Egypt. For the latitude of Giza at the time of construction of the Great Pyramids, the pair of stars proposed by K. Spence—Mizar (ζ Ursae Majoris) and Kochab (β Ursae Minoris)—could have been observed with Mizar at lower position (and Kochab at the upper) in summer and, reciprocally, with Mizar at the upper position (and Kochab at the lower) in winter. A problem specific to Spence's stars is as follows: for the pair Kochab and Mizar one should observe an arc of ca. 42° above the horizon.[6] To visualize the vertical alignments of stars K. Spence used the astronomical software SkyMap Pro6; recent comparison of astronomical software currently available for archaeoastronomical applications demonstrated, however, noticeable differences between the programs and following the recommendation of De Lorenzis and Orofino (2018), the freely available software *Cartes du Ciel* based upon the latest precession theory (Vondrák et al. 2011) will be used throughout this paper.[7]

The vertical alignment of the two stars as it was seen at the latitude of Khufu's pyramid at 14.01.2478 BC is displayed in Fig. 2.

[6] K. Spence proposed a sort of giant *merkhet* with a plumb on a long string. Such a construction is not documented in the ancient Egyptian sources.

[7] Parkerson (2019). Note that the nutation is neglected in the program for the time period before 500 BC. The nutation series do not contain by definition secular terms and should contribute to our calculations below the level of 0.5′. Another unknown constraint of the ancient observations—the effects of refraction—does not noticeably influence the azimuths of the vertical alignments of stars: refraction mainly acts by 'raising' an observed object.

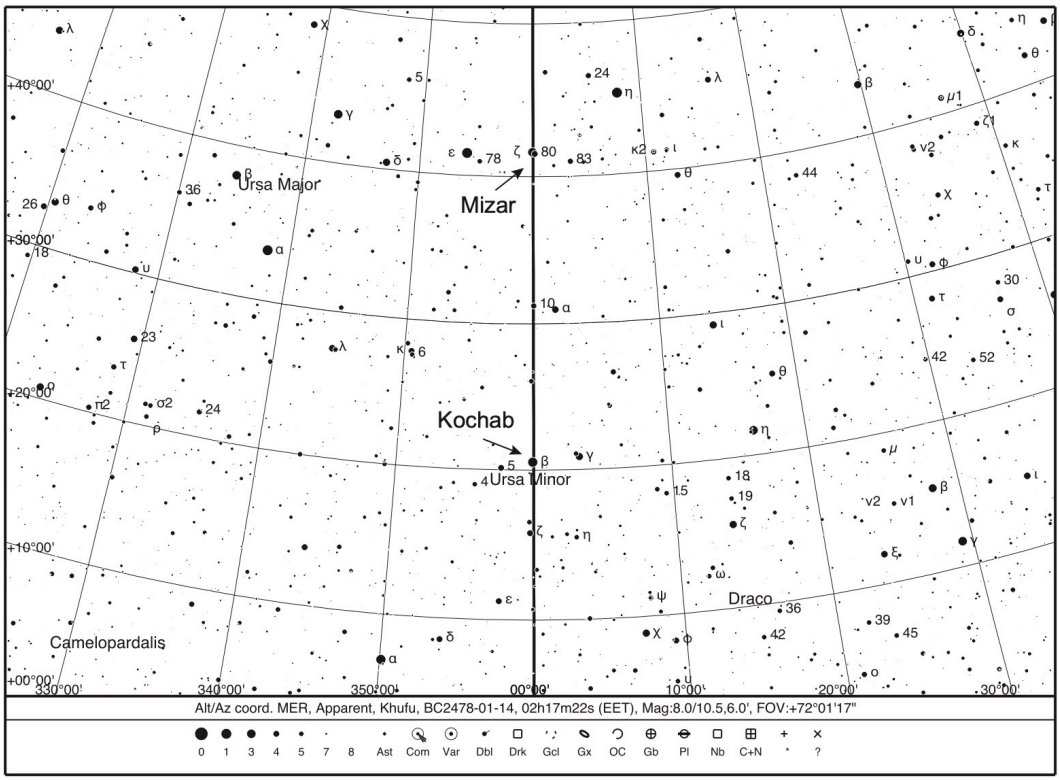

Fig. 2: Vertical alignment of Mizar and Kochab at the latitude of Khufu's pyramid on January 14, 2478 BC. Map produced on Cartes du Ciel.

Azimuths of the east sides of the pyramids given in Table 2 vs. azimuths of the vertical through Mizar and Kochab are shown in Fig. 3 for the chronology used by K. Spence. Here, the line 'a' represents the azimuths of the alignment of Mizar at the upper position with Kochab at the lower position. As shown in Fig. 3, the trendline 'a' through the azimuths of the vertical alignment of Kochab and Mizar has a gradient similar to the gradient of the trendline 'c' through the azimuths of the east sides of the pyramids, and that was the crucial argument of K. Spence for the choice of this pair of stars. The deviations between the three trendlines lines can be interpreted in different ways. First, as a systematic error of the method in the range of about ±25'—such an accuracy, close to the apparent size of the Sun, is what we would expect from all we know about the surveying techniques in ancient Egypt. It is the high *precision* of the orientation of the 4th Dynasty pyramids to true north which tempts us to assume also the high *accuracy* of their orientation towards the target of observation. This precision forces us to infer that the method of the orientation of the pyramids had a very small systematic error—this interpretation was silently taken by K. Spence, followed by many other scholars.[8] K. Spence proceeded in the following way: assuming that the chronology for line 'a' can be considered fixed, the difference between the points where the two lines—'a' and 'c'—crosses the line y = 0 was used as a temporal shift to align these two lines. In this way, K. Spence proposed that the historical chronology of the Old Kingdom should be changed by lowering of von Beckerath's estimate by a further 74 years".[9]

[8] Rawlins and Pickering 2001; Belmonte 2001; Magli 2003; Puchkov 2019.
[9] Spence 2000: 324. According to our calculations, the line 'a' crosses the zero line of the y-axis at 2464 BC.

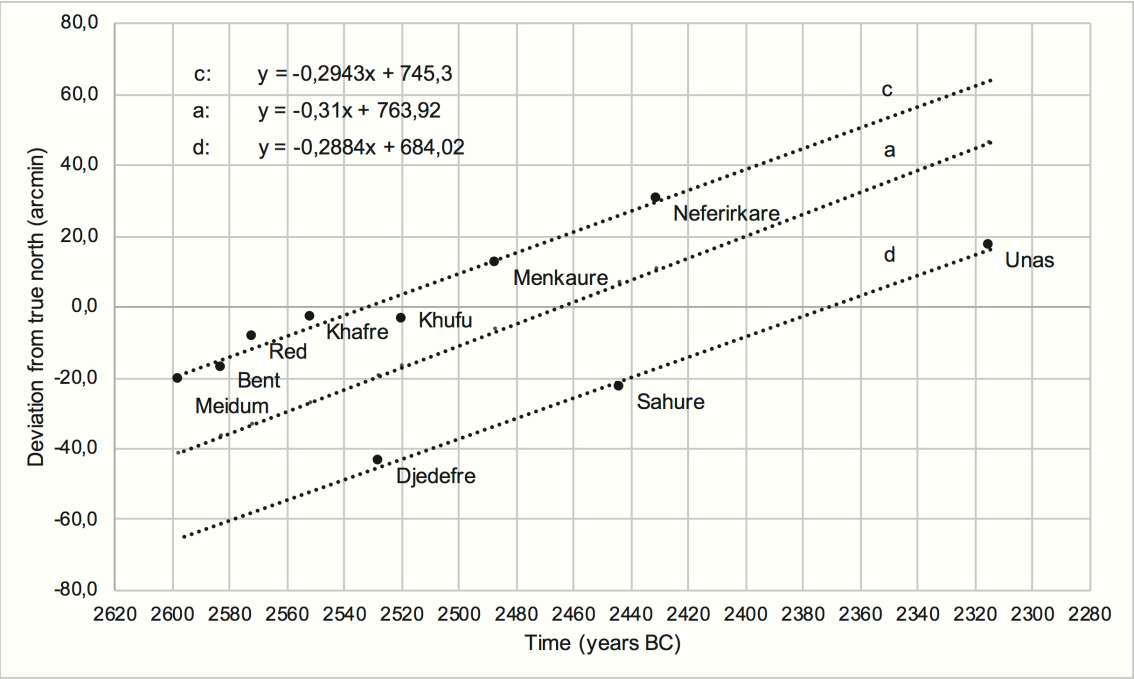

Fig. 3: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the vertical alignment of Mizar and Kochab (von Beckerath's chronology using 46 years for the duration of Snofru's reign). The dashed line 'a' is a trendline through the points corresponding to the azimuth of the vertical alignment of Mizar at the upper with Kochab at the lower position.

As we have noticed, after the proper calculation of the azimuths of the stellar vertical,[10] Khufu's pyramid is no longer the best oriented towards this target, although it is best oriented towards true north. In fact, it is the pyramid of Khafre which seems now to be best oriented towards the stellar vertical proposed by K. Spence. The known problem of its orientation, however, is that it is identical with that of Khufu's pyramid in spite of more than 30 years between the accession dates of the two kings. It is not only practically the same orientation of the sides of these two pyramids as shown in Table 1, but also the descending passages have a strikingly identical orientation: -5.6′ vs -5.8′, respectively (Petrie 1883, 58, 104). The proposed hypotheses to explain this were copying of alignment,[11] or that two pyramids were laid down simultaneously (Khufu's double project).[12] As a result of the chronological correction, the deviations in the orientation of three other pyramids—Djedefre's, Sahure's and Unas's—will increase and cannot be attributed to the same method anymore.

The starting chronology for Spence's algorithm was that of von Beckerath with 46 years for Snofru's reign. The same procedure of 'chronological correction' can also be applied to the dates of the alignment ceremonies corresponding to Show's chronology. However, in this case the solution will become more ambiguous, because the deviations of Djedefre's, Sahure's, and Unas' pyramids be smaller than those of the pyramids along the main trendline and it would be more natural to base the chronological correction upon the data for this group (Tupikova (2022, 21, Fig.7).), thus leaving aside the challenging question of the orientation of the pyramids of the 4th Dynasty.

[10] Astronomical modelling of the 'simultaneous transit method' was included in the original paper of K. Spence with a mathematical error in calculation of the azimuths noticed by D. Rawlins and K. Pickering (2001, 699).
[11] Rawlins 2003: 3.
[12] Shaltout, Belmonte, and Fekri 2007; Magli 2005; Magli and Belmonte 2009. That is why the data for Khafre's pyramid, although shown in the graphs in this text, will not be used in the calculation of trendlines.

An alternative pair of stars—Phecda (γ Ursae Majoris) and Megrez (δ Ursae Majoris)—was proposed by J. A. Belmonte (2001); the observational situation at Giza in 2571 BC is shown in Fig. 4. In contrast to the stars chosen by Spence (2000), these stars are aligned vertically, both being close to the lower or upper culmination.

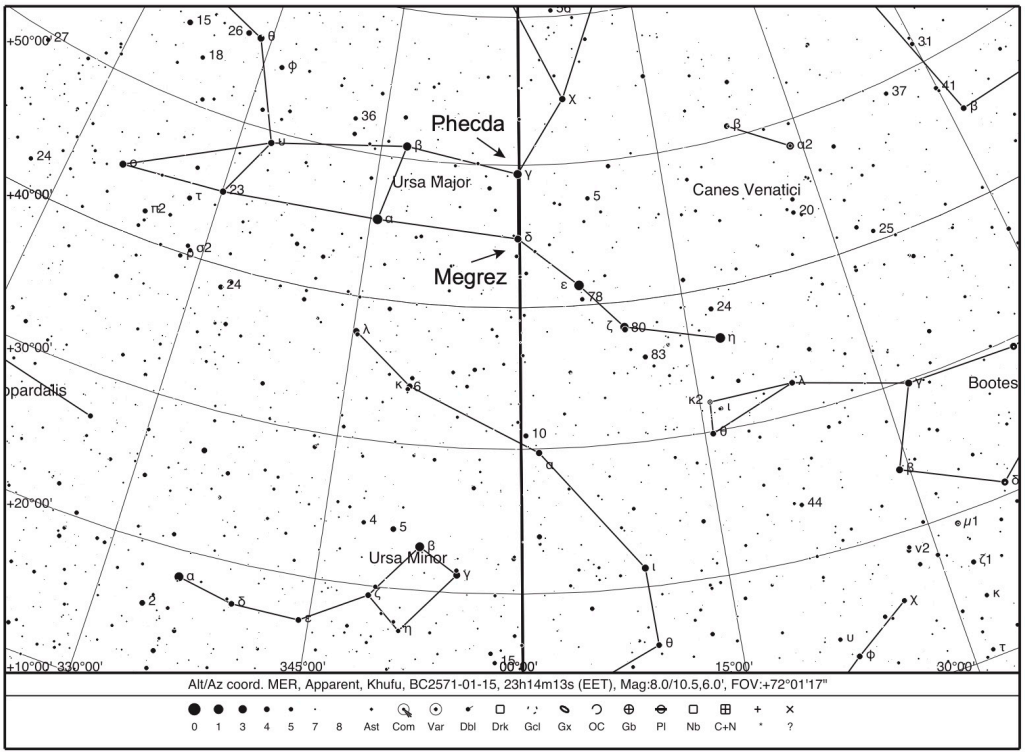

Fig. 4: Vertical upper alignment of Phecda and Megrez at the latitude of Khufu's pyramid on January 15, 2571 BC. Map produced on *Cartes du Ciel*.

J. A. Belmonte based his method upon the same assumption as K. Spence: that the real aim of the pyramid orientation was the North Celestial Pole. According to this author, Khufu's pyramid could have been aligned between 2571 and 2565 BC, at the time when Phecda and Megrez were at their upper vertical alignment; that gives "dates for Khufu's ascension to the throne that are just in the middle of the highest and lowest chronologies accepted today by Egyptologists"(Belmonte 2001, S12).[13] With the actual precession theory, we can state that for the location of Khufu's pyramid, the vertical through Phecda and Megrez crossed the meridian around 2552–2551 BC. We have checked Belmonte's proposal for the chronologies given in Table 2 and can conclude that for all data the trendlines drawn through the azimuths of the vertical through Phecda and Megrez does not match the trendlines in the orientation of the pyramids (for discussion see Tupikova 2022, 22–27). Fig. 5 displays, for example, the results of Belmonte's method for von Beckerath's chronology.

[13] The author considers in his text the high chronology according to J. Malek (Shaw 2000, 89–117) who assumes for Khufu's reign 2589–2566 BC and low chronology according to Baines and Malek (1980, 36) with the dates of 2551-2528 BC.

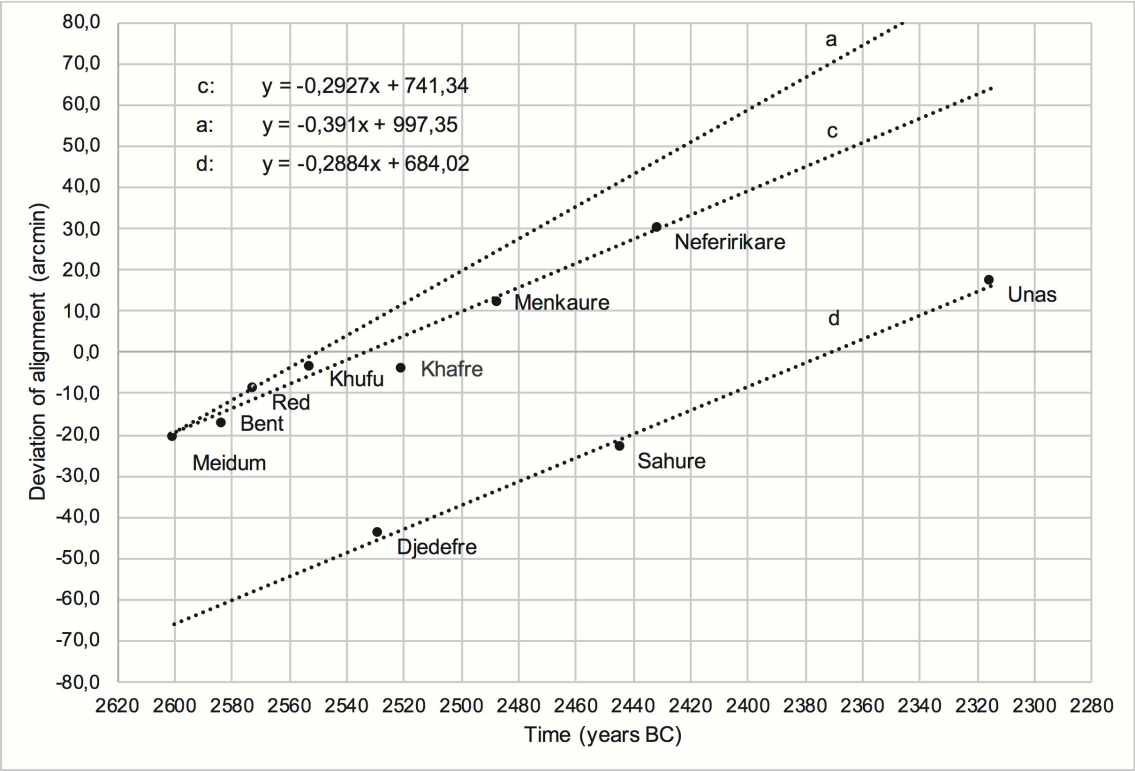

Fig. 5: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the vertical alignment of Phecda and Megrez (von Beckerath's chronology). The dashed line 'a' is a trendline through the points corresponding to the azimuth of the upper vertical alignment of Phecda and Megrez.

The choice of Phecda and Megrez has the same problem as the pair proposed by K. Spence: to explain the trendline in the orientation of the pyramids, one should consider the vertical alignment of stars at their upper position, i.e. with Phecda at almost 50° and Megrez at about 45° above the horizon.

The question was often asked: on what grounds the ancient Egyptians concluded that a chord passing through this pair of stars would also pass through the Celestial Pole? And on what grounds they made the choice of a particular pair of stars?

3 Vertical alignment of stars: a new proposal

We assume that some other ideas played a role in the orientation of pyramids—e.g. a vertical alignment of stars might have been considered as a sort of a stellar elevator to the celestial realm, the king's final destination as stated in the *Pyramid Texts*:[14] "A stairway to the sky is set for you among the Imperishable Stars [Circumpolar stars]." A remarkable geometriclal configuration of stars is known for us as the constellation Little Dipper where two sides of the Dipper are seemingly arranged as almost parallel lines. Every of these sides can also be observed as a vertical configuration at low altitude. The moment of the vertical alignment of two stars of one of the side of the Dipper—Kochab and ζ UMi—is shown in Fig. 6.

[14] Faulkner 1966, 156, Nt. 773–74.

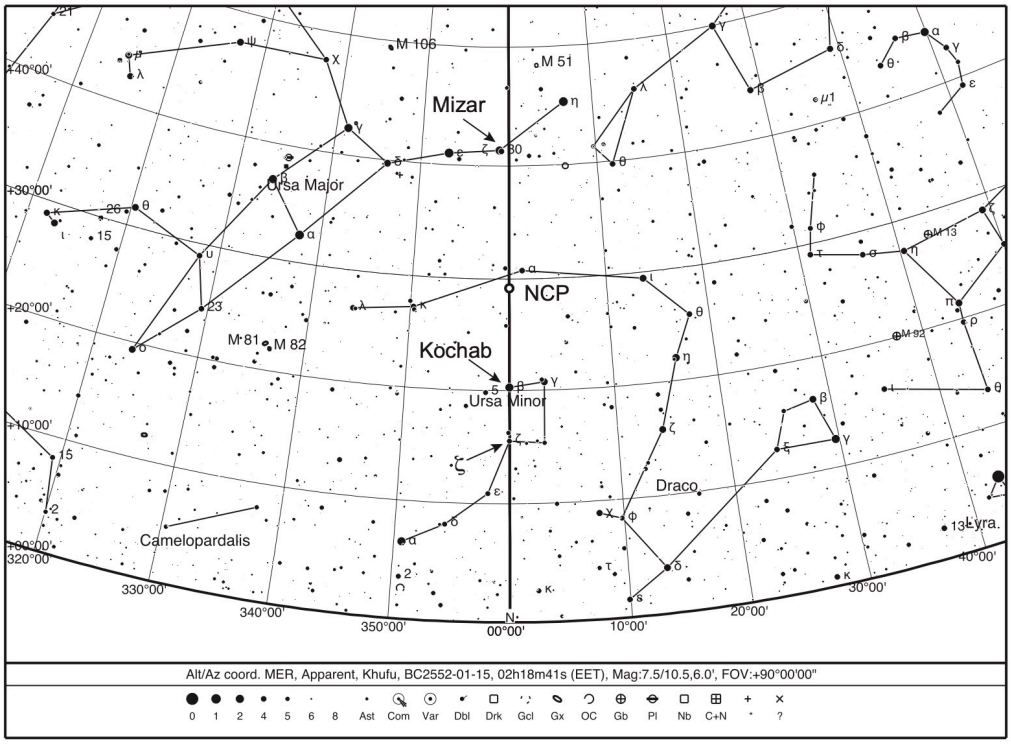

Fig. 6: Vertical lower alignment of β UMi (Kochab) and ζ UMi at the latitude of Khufu's pyramid on January 15, 2552 BC. Map produced on *Cartes du Ciel*.

We have checked the azimuths of this vertical configuration against the chronologies given in Table 2 and obtained the best match for von Beckerath's chronology; the results are shown in Fig. 7. The precision of this orientation calculated for every pyramid along the main trendline separately shows no secular trend over time and stay at a surprisingly high level: the mean deviation of the orientation of the pyramids towards the stellar vertical is ca. -4′ with a standard deviation of 2′ (Tupikova 2022, Fig. 30). Such a small deviation is, in fact, close to the limit of naked-eye observations.

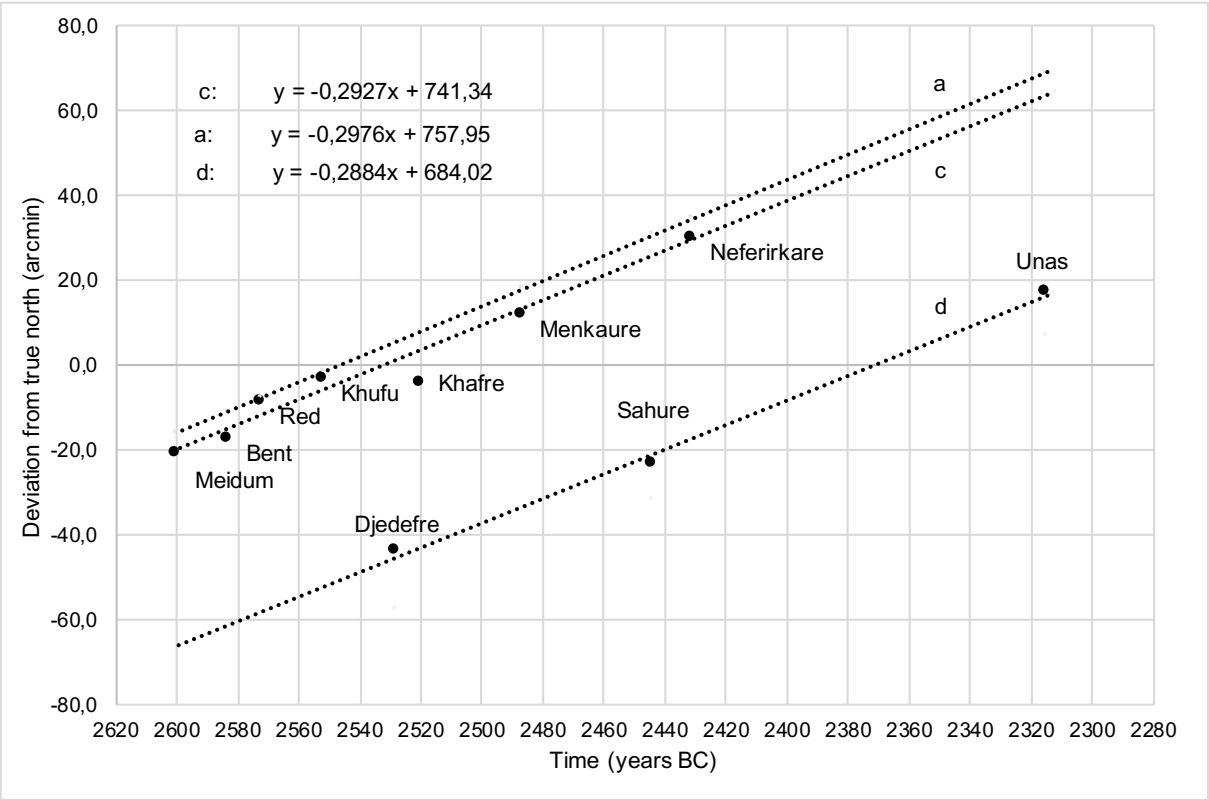

Fig. 7: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the lower vertical alignment of Kochab and ζ UMi (von Beckerath's chronology). The dashed line 'a' is a trendline through the points corresponding to the azimuths of their alignment.

Even better match can be obtained for the west sides of the pyramids where only scarce measurements ar available. As shown in Fig. 8, the precision of this orientation is very impressive and cannot be questioned for the Meidum, Bent and Khufu's pyramids. The only visible deviation from the trendline 'a' is for Menkaure's pyramid. One should take into account, however, that because the foundation of the pyramid was covered with rubble, Nell and Ruggles (2014) were only able to survey the alignment of courses of stones on the pyramid itself. The results for two block courses (9th and 11th) were given in their Table 5c with orientations of 29.5′ and 19.7′, correspondingly (the figure of 25′ given in our Table 1 is the mean value of these estimations). For the figure of 19.7′, the azimuth of the west side of Menkaure's pyramid would lie exactly on the line 'a'; this position is marked at Fig. 8 with a cross.[15] In our opinion, although the data for the west sides of the pyramids are scarce, these calculations strongly support the idea that the *Stretching of the Cord* ceremonies started with the west sides and usually with the north-west corner.[16]

[15] On what grounds this change in the orientation was made is not clear. Did the ancient architects notice that in the process of the construction the initial orientation was distorted, and try to correct it? If so, the initial orientation was close to the orientation of the 11th course.

[16] The corresponding inner angle of the north-west side of the pyramid would also best match the right angle (Tupikova 2022, Appendix C). See also Spence 2000: 321.

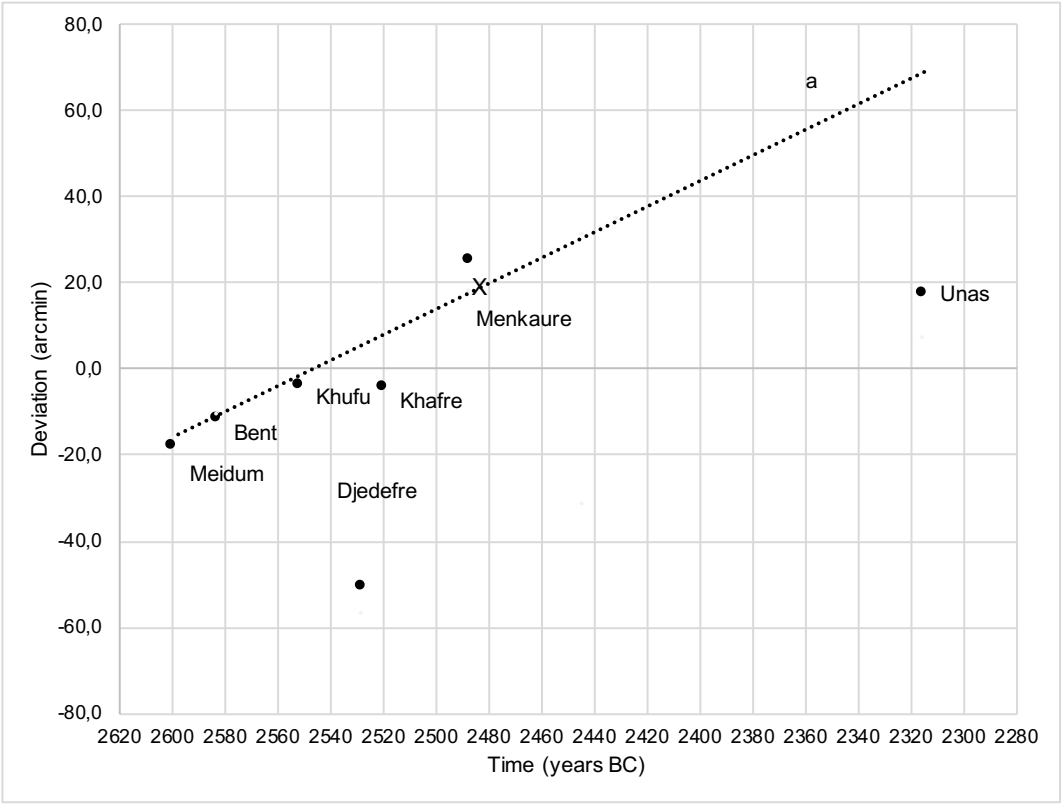

Fig. 8: Deviations of pyramid orientations (west sides) from true north over time vs. azimuth of the lower vertical alignment of Kochab and ζ UMi (von Beckerath's chronology). The dashed line 'a' is a trendline through the points corresponding to the azimuths of their alignment.

An important advantage of our proposal is that the vertical alignment of Kochab and ζ UMi matching the trend in the orientation of the pyramids could have been observed at a lower altitude, and this fact makes our method the only solution advanced so far which eliminates the problem of the high-altitude observations; in this case, a standard old Egyptian astronomical instrument, a *merkhet*[17] supplied with a plumb line, could approximately determine the moment of this vertical alignment, and the posts used in the *Stretching of the Cords* ceremony might have been used to additionally check the alignment and to fix the direction towards this stellar configuration on the ground.

Our results make any chronological correction unsubstantiated, and strongly confirm von Beckerath's low chronology for the dates of the 4th Dynasty with Stadelmann's modification for 48 years of Snofru's reign.

4 A possible iconographic evidence

As is well known, the Egyptian asterism *Meskhetiu* mentioned in the ceremony *Stretching of the Cord* was often represented in textual and pictorial evidence as a bull's foreleg, a stylised ovoid bull body (the tomb of Senenmut at Deir el-Bahari, ca. 1450 BC) or as a more realistic bull image (the burial chamber of Seti I in the King's Valley, ca. 1290 BC). In typical

[17] Two instruments preserved in the Egyptian Museum of Berlin have a different size but are marked according to the same scheme, although not very accurately (see Tupikova and Soffel, 2012). We can, therefore, assume that the size of the *merkhet* was not standardized and that bigger instruments possibly also existed.

representations, the constellation *Meskhetiu* was not displayed with any special vertical or horizontal arrangement of stars.[18] There exists, however, another important asterism mentioned in the Pyramid Texts as the *Mooring Post* which is usually represented as two daggers (often one in the form of a small crocodile, and occasionally only one dagger is displayed) in the hands of the most remarkable stellar constellation in the astronomical diagrams, the *Female Hippopotamus*. These daggers, variously interpreted in modern literature, are depicted as parallel *vertical* objects; in some iconographic traditions, a chain connects one of the daggers with the tail of the Bull (Fig. 9). We propose here the interpretation of the *Mooring Posts* as two sides of the Little Dipper represented by the pair Kochab and ζ UMi, and by the pair γ and η UMi at the moment when they were aligned vertically as shown in Fig. 6; in our opinion, vertical alignment of Kochab and ζ UMi might have been considered to be a sacral way to the the king's final destination in the asterism *Meskhetiu*:[19]

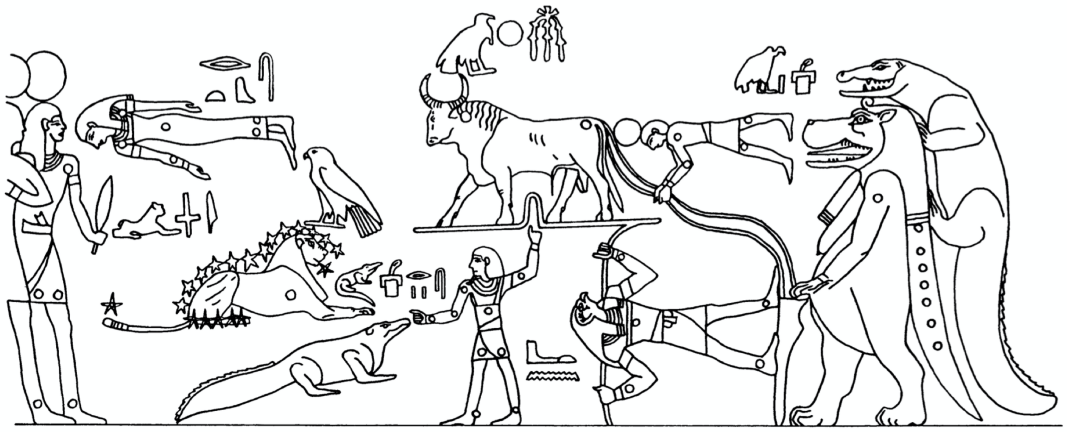

Fig. 9: Ceiling detail, tomb of Seti I, Sarcophagus Chamber C. After Wilkinson 1991, Fig. 2.[20]

The vertical alignment of the pair Kochab and ζ UMi (Fig. 6) is exactly this position which corresponds to the upper position of the Bull's Foreleg as displayed in the preserved iconographic evidence (such as ceiling details in the Temple of Ramses II or in the tomb of Seti I); it is shown schematically in Fig. 10 relative to the northern panel of Senenmut's astronomical diagram.[21]

[18] Different traditions of representation of the *Meskhetiu* are discussed by Neugebauer and Parker (1969, III, 183–189).

[19] In the Pyramid Texts of Pepi I, the king is addressed as "you of the mooring, who came from the *Mooring Post*" (Allen 2005, 102, Pepi I Spell 14). See also Merenre Spell 399 (Allen 2005, 235).

[20] Fig. 9 is given by R. H. Wilkinson with the reference to Neugebauer and Parker (1969, III) and reproduced by permission of the American Research Center in Egypt, Inc. (ARCE).

[21] After C. Wilkinson (The Met's Open Access).

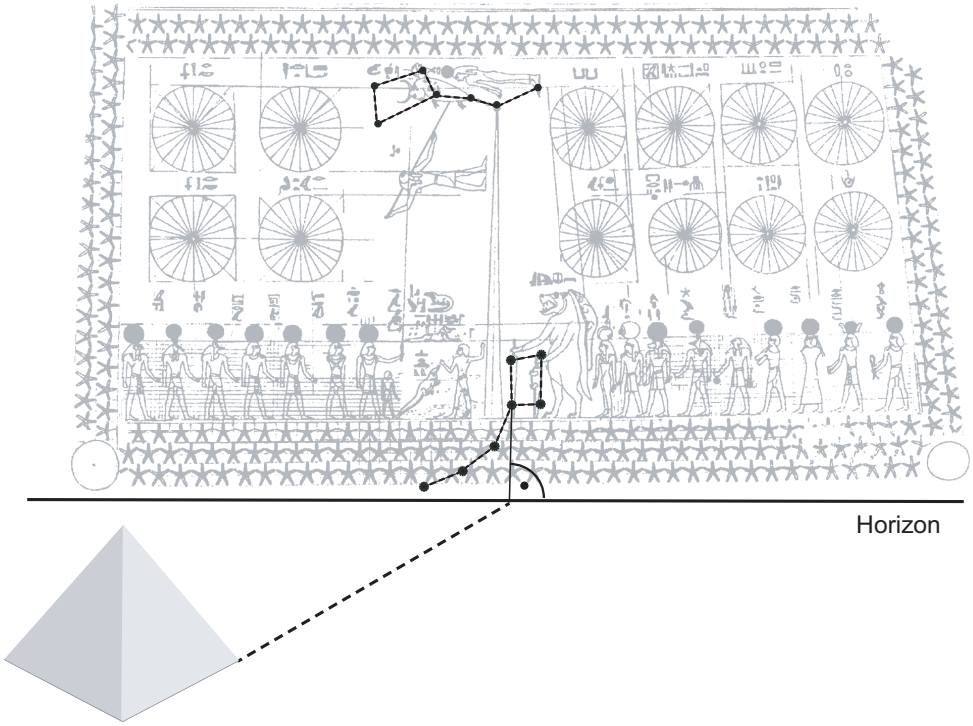

Fig. 10: Symbolic interpretation of the observation of the vertical alignment of Kochab and ζ UMi against the northern panel of Senenmut's astronomical diagram (not to scale).

In the northern panel of Senenmut's astronomical diagram where three stars in the bull's tail are displayed in an inclined line, the third star is colored in red and encircled by a red ring; it is tempting to assume that this third star might be identical with Mizar and that this star was an actual target of king's final destination in the celstial realm (for the discussion see Tupikova 2022. 32—40).

5 Horizontal alignment of stars

In their text "Astronomical orientation of the pyramids" published as an answer to the paper of K. Spence, D. Rawlins and K. Pickering firstly proposed to use a horizontal alignment of stars to explain the orientation of pyramids. The authors noticed that two stars—Thuban and 10 Draconis—were equidistant from the Pole in 2627 BC and that "when both stars were at the same altitude, north was the direction bisecting them" (Rawlins and Pickering 2001, 699). Although relatively faint (4.5-5.0 mag), 10 Draconis is recorded in all large naked-eye star catalogs but without its hieroglyphic identification and without chronologies supporting the corresponding dating of the pyramids (which implied dates of 2638 BC for Khufu's pyramid), the proposal has not gained any support in Egyptology. The same question can be posed here: on what grounds did the ancient Egyptians conclude that Thuban and 10 Draconis could be used at this particular position to mark the direction towards the Celestial Pole?

Recently, the text entitled "Stretching of the Cord Ceremony for Astronomical Orientation of the Old Kingdom Pyramids" was released at the internet portal *academia.edu* by A. Puchkov (2019). The main idea of the author was that the orientation of the pyramids was performed in the moment when two stars—Dubhe and Alkaid—in the constellation Big Dipper were aligned horizontally; at this moment, the direction towards the star Thuban was used to align the sides of the pyramids along the main trendline. Thus determined, the trendline of the azimuth of Thuban was compared by the author with the trendline of the azimuths of the

pyramids to link the absolute and relative chronologies; the proposal of A. Puchkov demands "a shift in the dates of the Old Kingdom by a little more than two centuries to the past" (Puchkov 2019: 19). As we have checked, for the new proposed chronology, the precision of the orientation of the pyramids towards Thuban reveals a secular trend implying decreasing precision of the alignments that is difficult to explain (Tupikova 2022, Figs.17—18). This proposal has also a problem of observation of the corresponding stellar configuration at high altitudes.

The alignment of Dubhe and Alkaid is not the only possible horizontal alignment in the constellation *Big Dipper*. As we have noticed, two other prominent bright stars—Alioth and Mizar—aligned horizontally tend to support von Beckerath's chronology with Mizar being a target of observations at the moment of such horizontal alignment (Tupikova 2022, Figs. 19—21). According to our calculations, in the time of the Old Kingdom, the azimuth of Mizar at the moment of its horizontal alignment with Alioth was very close to the azimuth of the vertical alignment of Kochab and ζ UMi; the two directions practically coincided in 2562 BC (Tupikova 2022, Fig. 31).

Could this remarkable astronomical coincidence have been used as a crossover check for the orientation of pyramids? Could this alignment also provide a unique opportunity to orient astronomically both the east and west sides of some pyramids in the course of one night?

Conclusion

The principal distinction in our approach to the problem of the temporal trend in the orientation of the Old Kingdom pyramids is of an epistemological character: whereas the majority of previous scholars assumed the intended orientation of the pyramids to be the North Celestial Pole and looked for the stars which could have helped to mark its position at the time in question, we assume instead that the ancient architects oriented the pyramids using remarkable stars in remarkable geometrical congifigurations.

In the time of the Old Kingdom, two astronomical events might have been observed in close temporal proximity—the horizontal alignment of Mizar with Alioth and the vertical alignment of Kochab with ζ UMi. As we have shown, these events had also a close azimuthal proximity: provided that the direction towards Mizar at the moment of its horizontal alignment with Alioth was fixed, about 16 minutes later almost the same direction was marked by the vertical alignment of Kochab and ζ UMi. It was shown that the azimuths of these two different stellar alignments would match the trend in the orientation of the pyramids with remarkable and similar precision.

Of course, the evidence discussed in this text is not a strong mathematical proof, and the comparison of the graphs cannot serve as a final argument to reject the methods proposed by other authors. But our results match the main trendline in orientation of the pyramids with such a compelling precision for the widely agreed upon chronology that the probability of a chance match is approximately the same as the probability of a chance temporal trend in the orientation of the pyramids.

The azimuths of the alignments of stars depend not only upon their right ascensions and declinations, but also upon the geographical position of observation. That the proposed alignments occured at the time of the construction of the Old Kingdom pyramids close to true north is, in our opinion, a fortuitous event which is mainly responsible for the remarkable northern orientation of these pyramids.

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