General relativity tests in a dynamical model of the Solar system

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Outline

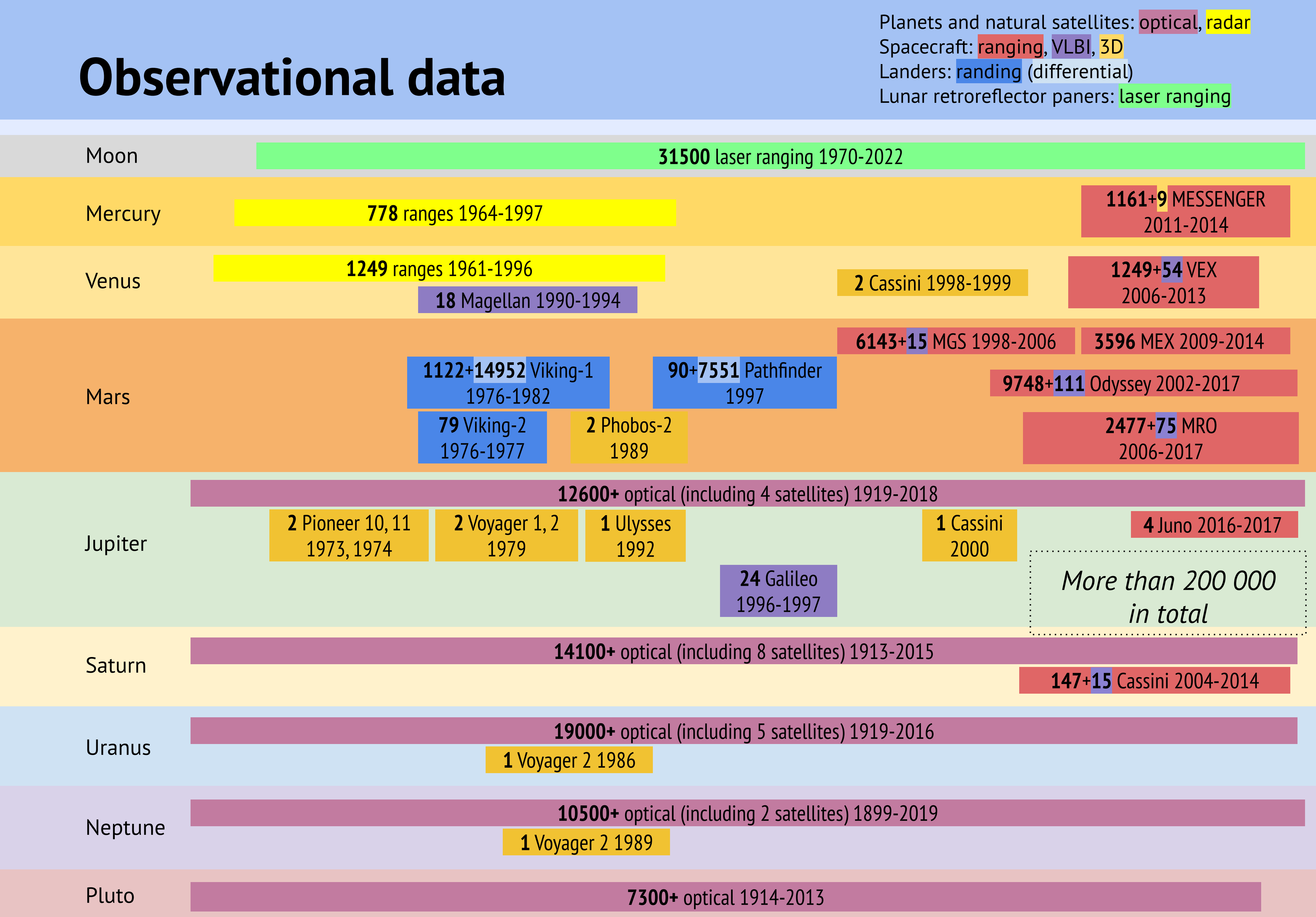
Checking for two deviations from GR in lunar-planetary numerical ephemeris:

- ➢ Equivalence principle (by Earth-Moon-Sun dynamics)
  - ○ Observational data: lunar laser ranging (LLR)
- ➢ Change rate of gravitational constant G (by planetary dynamics)
  - ○ Observational data: Earth-Mars, Earth-Mercury ranges to planet orbiters, other ranging data, optical data

Dynamical model of the Solar system includes:

- ○ Point-masses interaction according to GR
- ○ Sun's oblateness (J2) and Lense-Thirring acceleration
- ○ Kinematic equations of the Earth's rotation and tides, acceleration and torque from non-spherical gravity up to degree 6
- ○ Elastic Moon with tides and liquid core, non-spherical gravity up to degree 6

Observational data



Planetary equations

- ➢ Einstein-Infeld-Hoffmann relativistic equations for point masses (Sun, Moon, planets, Pluto, 5 asteroids)
- ➢ Acceleration from solar oblateness
- ➢ Lense-Thirring acceleration

$$\vec{a}_A = \sum_{B\neq A} \frac{Gm_B \vec{n}_{BA}}{r_{AB}^2}$$
$$+ \frac{1}{c^2} \sum_{B\neq A} \frac{Gm_B \vec{n}_{BA}}{r_{AB}^2} \left[v_A^2 + 2v_B^2 - 4(\vec{v}_A \cdot \vec{v}_B) - \frac{3}{2}(\vec{n}_{AB} \cdot \vec{v}_B)^2 \right.$$
$$\left. - 4 \sum_{C\neq A} \frac{Gm_C}{r_{AC}} - \sum_{C\neq B} \frac{Gm_C}{r_{BC}} + \frac{1}{2}((\vec{x}_B - \vec{x}_A) \cdot \vec{a}_B) \right]$$
$$+ \frac{1}{c^2} \sum_{B\neq A} \frac{Gm_B}{r_{AB}^2} \left[\vec{n}_{AB} \cdot (4\vec{v}_A - 3\vec{v}_B) \right] (\vec{v}_A - \vec{v}_B)$$
$$+ \frac{7}{2c^2} \sum_{B\neq A} \frac{Gm_B \vec{a}_B}{r_{AB}} + O(c^{-4})$$

$$f_x = -\frac{3}{2}\mu J_2 \frac{R^2}{r^5} x \left(1 - 5\frac{z^2}{r^2}\right)$$

$$f_y = -\frac{3}{2}\mu J_2 \frac{R^2}{r^5} y \left(1 - 5\frac{z^2}{r^2}\right)$$

$$f_z = -\frac{3}{2}\mu J_2 \frac{R^2}{r^5} z \left(3 - 5\frac{z^2}{r^2}\right)$$

$$\ddot{\mathbf{r}}_i^{\mathrm{LT}} = \frac{2}{c^2} G S_{\mathrm{Sun}} \frac{1}{r_{iS}^3} R_{\mathrm{Sun}} \left(\dot{\mathbf{r}}_{iS} \times \mathbf{z} + 3\frac{\mathbf{z} \cdot \mathbf{r}_{iS}}{r_{iS}^2} \mathbf{r}_{iS} \times \dot{\mathbf{r}}_{iS} \right)$$

Model of the Lunar Physical Libration

- ➢ Rigid body equations of rotation with an external torque
- ➢ Torque from point masses in lunar gravitational field: Earth, Sun, Venus, Mercury, Mars, Jupiter
- ➢ Figure-figure torque between Earth's J2 and Moon
- ➢ Dynamic tensor of inertia: delayed dissipation from rotation and Earth tides
- ➢ Inertia and torque from inner rotating liquid core
- ➢ Torque from friction on the core-mantle boundary

$$\mathbf{r}_I = \mathcal{R}_z(-\phi_m)\mathcal{R}_x(-\theta_m)\mathcal{R}_z(-\psi_m)\mathbf{r}_{PA}$$

$$\begin{aligned}\dot{\phi}_m &= (\omega_{m,x}\sin\psi_m + \omega_{m,y}\cos\psi_m)/\sin\theta_m\\ \dot{\theta}_m &= \omega_{m,x}\cos\psi_m - \omega_{m,y}\sin\psi_m\\ \dot{\psi}_m &= \omega_{m,z} - \dot{\phi}_m\cos\theta_m.\end{aligned}$$

$$\dot{\boldsymbol{\omega}} = \mathbf{I}^{-1}\left\{\Sigma_i \mathbf{N}_{fig-pm} + \mathbf{N}_{fig-fig} - \dot{\mathbf{I}}\,\boldsymbol{\omega} - \boldsymbol{\omega}\times\mathbf{I}\,\boldsymbol{\omega}\right\}$$

$$\begin{aligned}\mathbf{N}_{fig-fig} = \frac{15\ \mu_e \mathcal{R}_e^2 J_{2e}}{2\ r_e^5}\{\ &(1-7\sin^2\phi)[\hat{\mathbf{r}}_\mathbf{e}\times\mathbf{I}\hat{\mathbf{r}}_\mathbf{e}]\\ &+2\sin\phi\ [\hat{\mathbf{r}}_\mathbf{e}\times\mathbf{I}\hat{\mathbf{P}}_\mathbf{e}+\hat{\mathbf{P}}_\mathbf{e}\times\mathbf{I}\hat{\mathbf{r}}_\mathbf{e}]\\ &-\frac{2}{5}[\hat{\mathbf{P}}_\mathbf{e}\times\mathbf{I}\hat{\mathbf{P}}_\mathbf{e}]\ \}\end{aligned}$$

$$\mathbf{I}_m(t) = \tilde{\mathbf{I}}_m - \frac{k_{2,M}m_E R_M^5}{r^5}\begin{bmatrix} x^2-\frac{1}{3}r^2 & xy & xz\\ xy & y^2-\frac{1}{3}r^2 & yz\\ xz & yz & z^2-\frac{1}{3}r^2\end{bmatrix} + \frac{k_{2,M}R_M^5}{3G}\begin{bmatrix}\omega_{m,x}^2-\frac{1}{3}(\omega_m^2-n^2) & \omega_{m,x}\omega_{m,y} & \omega_{m,x}\omega_{m,z}\\ \omega_{m,x}\omega_{m,y} & \omega_{m,y}^2-\frac{1}{3}(\omega_m^2-n^2) & \omega_{m,y}\omega_{m,z}\\ \omega_{m,x}\omega_{m,z} & \omega_{m,y}\omega_{m,z} & \omega_{m,z}^2-\frac{1}{3}(\omega_m^2+2n^2)\end{bmatrix}$$

$$\dot{\boldsymbol{\omega}}_m = \mathbf{I}_m^{-1}\left\{\sum_{A\neq M}\mathrm{N}_{M,figM-pmA} + \mathrm{N}_{M,figM-figE} - \dot{\mathbf{I}}_m\boldsymbol{\omega}_m - \boldsymbol{\omega}_m\times\mathbf{I}_m\boldsymbol{\omega}_m + \mathrm{N}_{cmb}\right\}$$

$$\dot{\boldsymbol{\omega}}_c = \mathbf{I}_c^{-1}\left\{-\boldsymbol{\omega}_m\times\mathbf{I}_c\boldsymbol{\omega}_c - \mathrm{N}_{cmb}\right\}$$

$$\mathbf{I}_c = \alpha_c C_T\begin{bmatrix}1-f_c & 0 & 0\\ 0 & 1-f_c & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\mathrm{N}_{cmb} = k_v(\boldsymbol{\omega}_c-\boldsymbol{\omega}_m)+(C_c-A_c)(\hat{\mathbf{z}}_m\cdot\boldsymbol{\omega}_c)(\hat{\mathbf{z}}_m\times\boldsymbol{\omega}_c)$$

Determined parameters

Solar system parameters

- Orbital elements of planets at epoch
- Gm of the Sun, the Moon, and 277 asteroids
- Gms of the Main asteroid belt and Kuiper belt
- Orientation angles of ephemeris frame to ICRF
- 13 parameters of rotation of Mars
- Earth/Moon mass ratio
- Two angles of solar equator plane
- Solar oblateness factor $J2_{\odot}$

Reduction parameters

- Corrections to reference surfaces of Venus, Mercury, and Mars
- 8 coefficients of topography of Mercury
- 5 phase corrections for outer planets
- Coordinates of 3 Martian landers
- Solar plasma delay factor
- Spacecraft ranging biases (MGS/Odyssey, MRO) for 15 DSN stations
- Corrections to spacecraft transponder delays
- Positions of lunar retroreflectors and lunar laser ranging reference points
- Corrections to LLR ranges (biases)

Lunar parameters

- Geocentric position and velocity of the Moon at epoch
- Lunar physical libration angles and their rates at epoch
- Ratios of the lunar moments of inertia
- Lunar core oblateness factor, core-mantle friction parameter
- Angular velocity of lunar core at epoch
- Lunar tidal delay
- Love number h_2

Special parameters (determined in solutions together with the normal parameters)

- Change rate of $Gm_{\odot}$
- m^g/m^i(Earth) − m^g/m^i(Moon)

About 500 parameters in total

Equivalence principle

Universality of free fall: $$\left(\frac{m^{\mathrm{g}}}{m^{\mathrm{i}}}\right)_M \stackrel{?}{=} \left(\frac{m^{\mathrm{g}}}{m^{\mathrm{i}}}\right)_E$$

Newtonian differential Earth-Moon acceleration in Sun's gravitational field:

$$\delta = \frac{m^{\mathrm{g}}}{m^{\mathrm{i}}} - 1$$

$$\begin{aligned}\boldsymbol{a}_{\mathrm{M}} - \boldsymbol{a}_{\mathrm{E}} = &-\frac{G\left[(1+\delta_M)m^{\mathrm{g}}_M) + (1+\delta_E)m^{\mathrm{g}}_E\right]}{r^3_{\mathrm{EM}}}\boldsymbol{r}_{\mathrm{EM}} \\ &+ Gm^{\mathrm{g}}_{\mathrm{S}}\left[\frac{\boldsymbol{r}_{\mathrm{SE}}}{r^3_{\mathrm{SE}}} - \frac{\boldsymbol{r}_{\mathrm{SM}}}{r^3_{\mathrm{SM}}}\right] \\ &+ Gm^{\mathrm{g}}_{\mathrm{S}}\left[\frac{\boldsymbol{r}_{\mathrm{SE}}}{r^3_{\mathrm{SE}}}\delta_E - \frac{\boldsymbol{r}_{\mathrm{SM}}}{r^3_{\mathrm{SM}}}\delta_M\right]\end{aligned}$$

$$\boldsymbol{a}^{\mathrm{EP}}_{\mathrm{ME}} = Gm^{\mathrm{g}}_{\mathrm{S}}\left[\frac{\boldsymbol{r}_{\mathrm{SE}}}{r^3_{\mathrm{SE}}}\delta_E - \frac{\boldsymbol{r}_{\mathrm{SM}}}{r^3_{\mathrm{SM}}}\delta_M\right] \approx Gm^{\mathrm{g}}_{\mathrm{S}}\frac{\boldsymbol{r}_{\mathrm{SE}}}{r^3_{\mathrm{SE}}}\left[\delta_E - \delta_M\right]$$

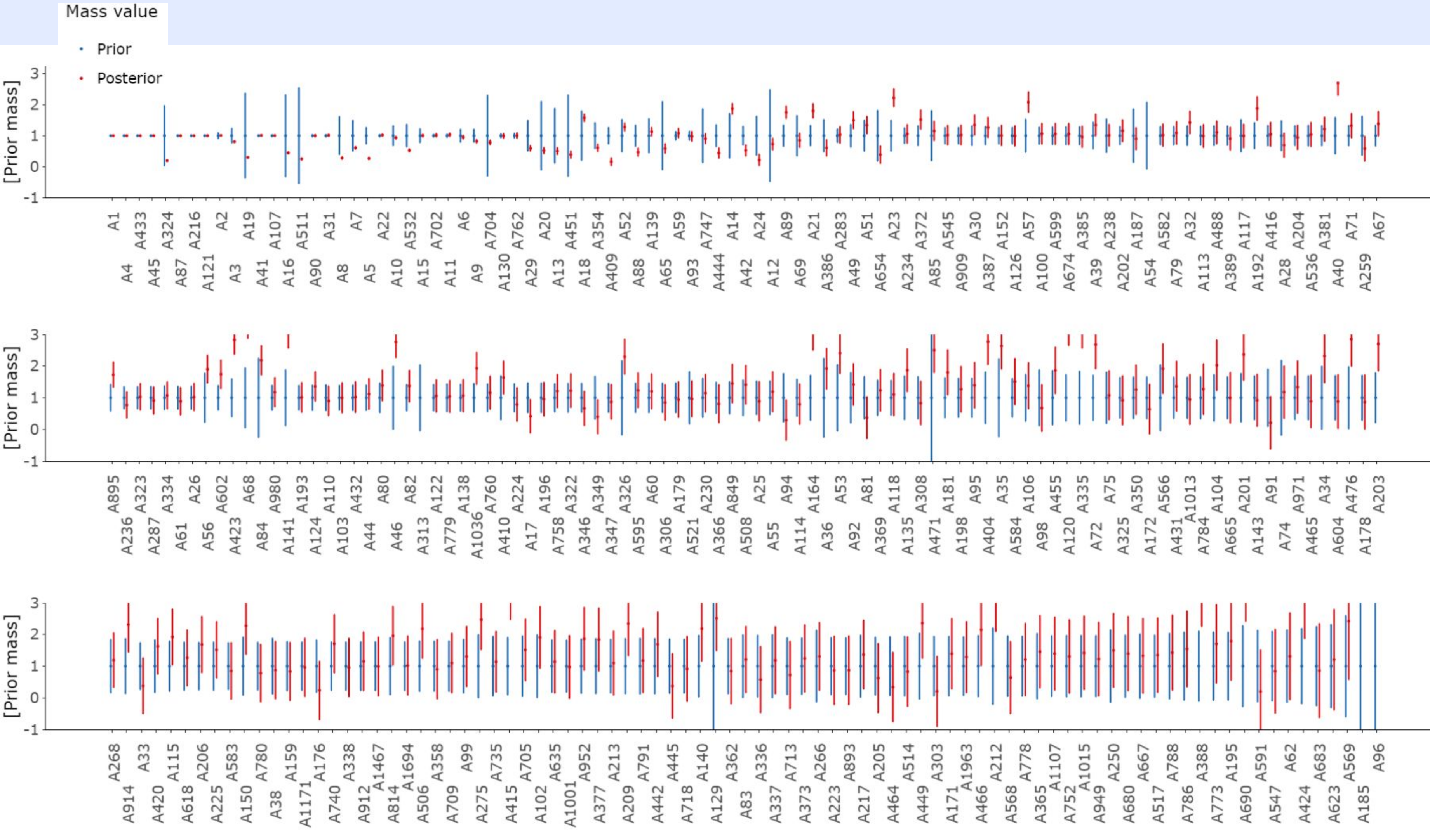
Lunar solution estimate: $\delta_E - \delta_M = (10 \pm 30) \times 10^{-15}$ (3σ error)

Conclusion: No sign of Earth's and Moon's accelerations in gravity of the Sun being different above the level of 3×10^{-14} (dimensionless units).

It is *probably* not possible to separate SEP and WEP using only the dynamics of the Solar system (see discussion in Viswanathan et al. 2018).

With WEP having been confirmed to better accuracy (Touboul et al. 2022), we can see the above as a bound for SEP.

Prior and determined asteroid masses

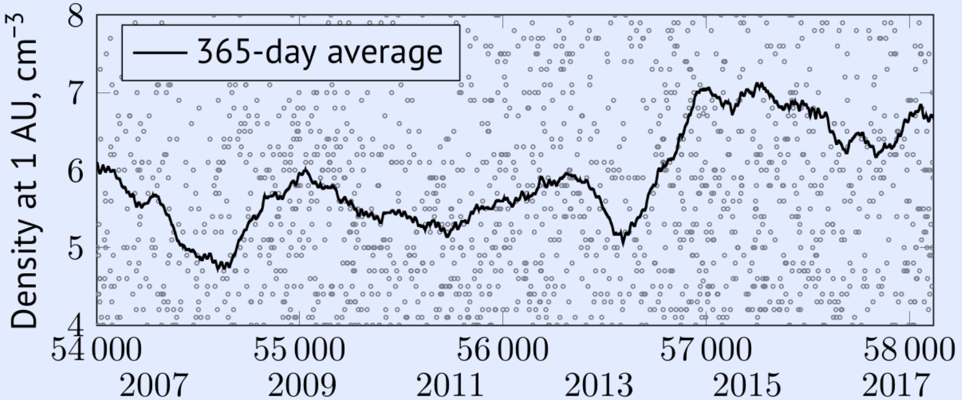


Modeling of delay in solar plasma

Spherically symmetric model

Ne ~ $1/r^2$

(Attempt to use the ENLIL solar wind data has not improved solution yet)

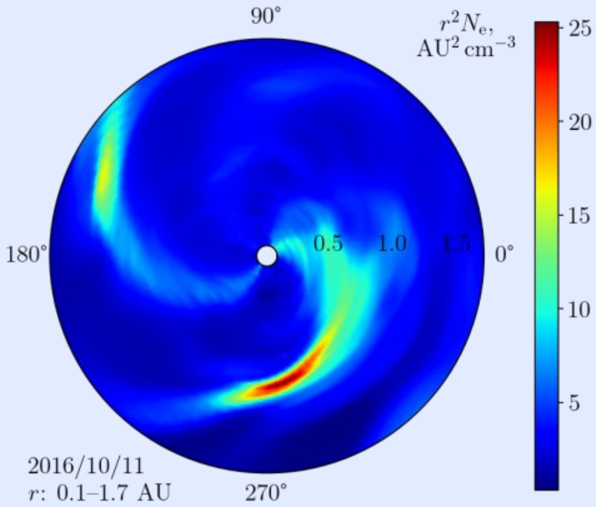


Medium-term variations of electron density in solar plasma

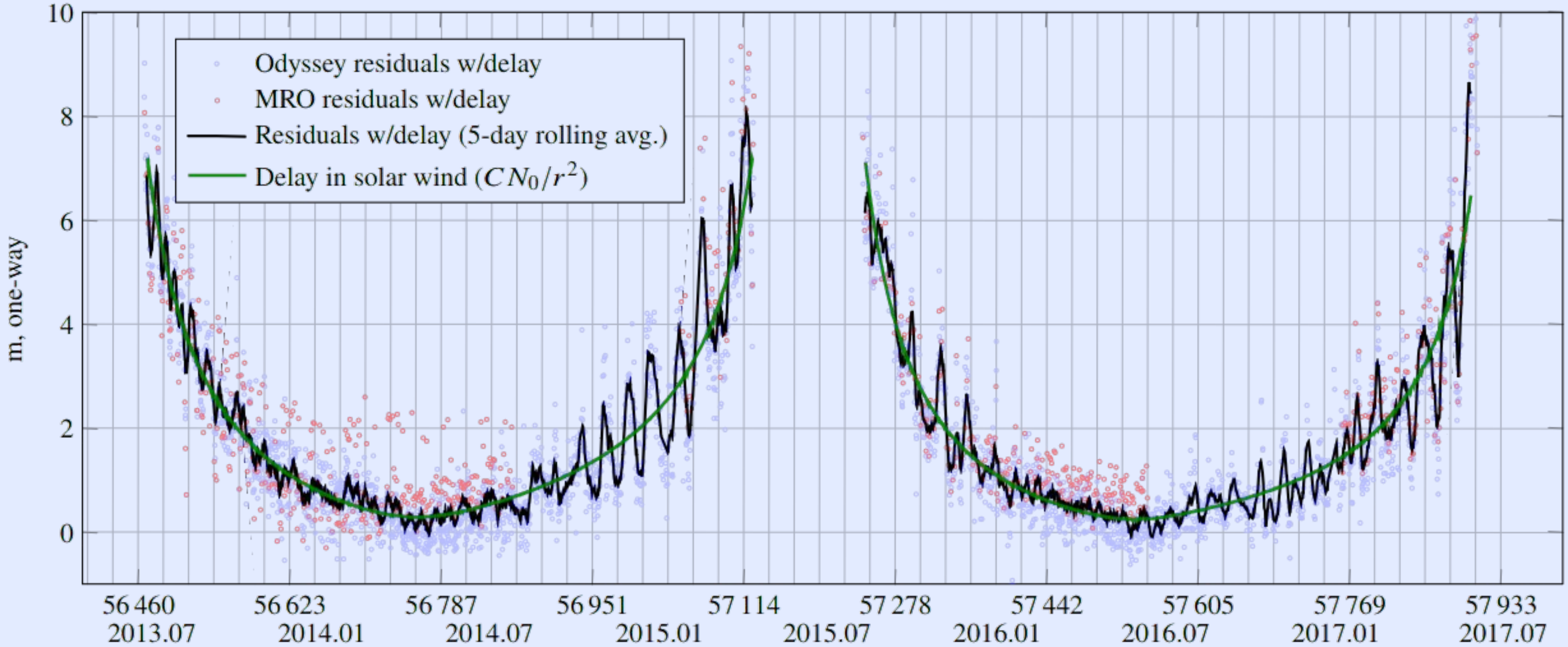
Electron density was calculated from OMNI data (King & Papatashvili, 2005) and then averaged to obtain yearly trends.

Single factor of solar plasma delay for 2009-2017

- The factor was regularized in the solution and came out equal to 1.034.
- The solar plasma delay correlates with $GM_\odot$ in the solution. The regularization allowed to improve the accuracy of $GM_\odot$ by ~12%.



Effect of solar plasma delay



Estimates of $\frac{\dot{G}}{G}$ using Solar System dynamics

$$\text{(estimate } GM_\odot \text{ and } G\dot{M}_\odot \text{ from LSM)}$$

$$GM_\odot(t) = GM_\odot + G\dot{M}_\odot t$$

$$\frac{G\dot{M}_\odot}{GM_\odot} = \frac{\dot{G}}{G} + \frac{\dot{M}_\odot}{M_\odot} = (-2.6 \pm 3.6) \times 10^{-14}$$

$$\frac{\dot{M}_\odot}{M_\odot} = \frac{1}{M_\odot}(\dot{M}_\odot^{\text{rad}} + \dot{M}_\odot^{\text{wind}} + \dot{M}_\odot^{\text{fall}})$$

$$\frac{\dot{M}_\odot^{\text{rad}}}{M_\odot} = (-6.7 \pm 1.8) \times 10^{-14}\,\text{yr}^{-1}\ (3\sigma)$$

Mass lost as a result of thermonuclear fusion. (IAU Resolution B3, 2015).

$$\frac{\dot{M}_\odot^{\text{wind}}}{M_\odot} = (-4.8 \pm 1.8) \times 10^{-14}\,\text{yr}^{-1}\ (3\sigma)$$

Mass carried away with the solar wind.
Data from the Ulysses spacecraft (NASA). Calculations from (Pitjeva et. al. 2021)

$$\frac{\dot{M}_\odot^{\text{fall}}}{\dot{M}_\odot} = (0.5 \pm 0.5) \times 10^{-14}\,\text{yr}^{-1}\ (3\sigma)$$

Asteroids falling onto Sun. Calculations from (Pitjeva et. al. 2021)

$$\frac{\dot{G}}{G} = (0.84 \pm 0.44) \times 10^{-13}/\text{yr} \qquad (3\sigma)$$

Secular change of G: discussion

Other modern estimates made outside the Solar system:

- (-6 ± 11) × 10^{-13} /yr (Zhu et al, 2015) [pulsar period]
- (-36 .. 45) × 10^{-13} /yr (Alvey et al, 2020) [baryon density from CMB data]
- (0.53 ± 0.60) × 10^{-13} /yr (Le, 2021) [quasar spectra]
- (0.0092 ± 0.028) × 10^{-13} /yr (Le, 2024) [quasar spectra]

Estimate by Solar system dynamics

- (-0.29 .. 0.46) × 10^{-13} /yr (Pitieva et al, 2021) [preceding work by same group]
- (0.84 ± 0.44) × 10^{-13} /yr (this work)

Low-frequency gravitational waves (Agazie et al. 2023) [pulsar timing arrays].

Nano-Hz frequencies (= periods of decades).