Adiabatic approximation in dynamical studies of exoplanetary systems in mean-motion resonance

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Abstract. If in a planetary system the ratio of the time periods of revolution of two planets around the host star is approximately equal to the ratio of two small integers, then such a situation is characterized as mean motion resonance (MMR). Available observational information indicates that MMR are quite common in exoplanetary systems. Analytical studies of MMR are carried out mainly within the framework of restricted or general three-body problem. In 1985, J. Wisdom proposed an approach that makes it possible to study the properties of the resonant motions of celestial bodies without any restrictions on the eccentricities and inclinations of their orbits. Since the application of this approach is associated with the construction of a special approximate integral of the problem (adiabatic invariant), it is often called the adiabatic approximation. We give a brief description of J. Wisdom's approach to the analysis of MMR and its subsequent development, illustrated by the results of the systematic use of this approach in our studies.

Introduction

Investigations of resonant motions in satellite and planetary systems are an important element in the study of their dynamical "skeleton", the properties of which determine the properties of many other physical processes in these systems. Classical approaches are focused primarily on the construction and study of periodic solutions of equations of motion (see, for example, [1]). In 1985, J. Wisdom, relying on the theory of adiabatic invariants (AI), showed how regions with chaotic dynamics are formed in the phase space of the three-body problem in the vicinity of resonant solutions [2]. A strict justification of Wisdom's constructions and estimates of the diffusion rate of AI at MMR 3:1 were given by A.I. Neistadt [3].

Adiabatic approximation in the study of resonances

The description of MMR in Wisdom's approach is completely equivalent to the picture of resonance effects given in textbooks on the modern theory of Hamiltonian systems (for example, [4]). The behavior of the system at MMR is characterized by the presence of dynamical processes with three time scales: "fast", "semi-fast" and "slow". A "fast" dynamical process is the orbital motion of resonant bodies. The "semi-fast" process is a variation of the resonant phase (a combination of mean longitudes, longitudes of periastrons and longitudes of the ascending nodes). The "slow" dynamical process consists of the secular evolution of the shape and orientation of the orbits of celestial bodies.

For a qualitative analysis of secular effects within the Wisdom's approach, double averaging of the equations of motion is applied. Averaging is carried out in two stages. The first stage consists of averaging over "fast" processes. After a series of transformations in the averaged equations, one can write down a subsystem that describes a "semi-fast" process, and a subsystem that describes "slow" processes. If we fix the values of the "slow" variables, the "semi-fast" system turns into an integrable Hamiltonian system with one degree of freedom (allowing a transition to "action-angle" variables). Averaging along its solutions of the right-hand sides of the equations of the "slow" subsystem completes the construction of evolutionary equations used to study secular effects.

In the general case, a "semi-fast" subsystem can be considered as a Hamiltonian system with slowly varying parameters, the role of which is played by slow variables. From this interpretation it follows that the "action" variable corresponding to this subsystem will be an approximate integral of the problem - an adiabatic invariant. Taking into account the existence of this AI, Wisdom characterized his approach as an adiabatic approximation.

An important difference between the adiabatic approximation and other approaches to MMR analysis is that it allows the consideration of possible transitions between resonant modes of motion (in which the resonant phase oscillates) and non-resonant modes (the resonant phase rotates). Each such transition is accompanied by a small quasi-random change in AI and a deviation of the true motion from what is predicted by the averaged equations. Repeated changes in the mode of motion lead to the diffusion of AI (in particular, we will see as the phase trajectory of the original system eventually fills a certain region in the phase space, called the region of adiabatic chaos).

Examples of the application of the adiabatic approximation in studies of MMR

J. Wisdom proposed his approach while studying MMR 3:1 in planar restricted three-body problem. The use of this approach to study other resonances required the introduction of various modifications taking into account their specifics.

To analyze MMR in exoplanetary systems, Wisdom's approach was adapted to the general three-body problem (more precisely, to the planetary variant of this problem when two low-mass bodies are moving in slightly perturbed Keplerian orbits around a significantly more massive body).

In our talk we present some properties of MMR 1:1 and 3:1 in the planar planetary problem, established using the Wisdom's approach [5]. Different scenarios of secular evolution were found and possible manifestations of chaotic dynamics were identified.

References

[1] A.D. Bruno, *Resricted three-body problem.* Nauka, 1990 (in Russian).

[2] J. Wisdom, *A perturbative treatment of motion near the 3/1 commesurability.* Icarus. 1985. V. 63. P. 272–286.

[3] A.I. Neishtadt, *Jumps of the adiabatic invariant on crossing the separatrix and the origin of the 3:1 Kirkwood gap.* Sov. Phys. Dokl. 1987. V. 32. P. 571–573.

[4] V.I. Arnold, V.V. Kozlov, A.I. Neishtadt, *Mathematical aspects of classical and celestial mechanics.* 3rd edition. Springer New York, 2006.

[5] V.V.Sidorenko, *Secular evolution of co-orbital motion of two exoplanets. Semi-analytical investigation.* Celestial Mechanics and Dynamical Astronomy. 2024. V. 136. 25 P.

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