Transit trajectories of ballistic capture near libration points for low-energy transfers

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Abstract. One of the approaches to increase the efficiency of interplanetary transfers is the use of low-energy transit orbits, which have a small energy change during transit from one massive body to another. The paper considers an approach to the design of transit trajectories of ballistic capture based on invariant manifolds of libration points L1 and L2. To study transit trajectories and capture duration, an elliptical 3-body problem and an ephemeris 4-body problem are used. The influence of the masses of massive bodies and the eccentricity of the orbit of a smaller body on the ballistic capture and its duration is analyzed. The use of the ΔV impulse to change the velocity of the spacecraft at the libration point to change the plane of the transit trajectory is considered.

Introduction

To improve transfers in a system consisting of several massive bodies, it is necessary to purposefully use the dynamics of three- and four-body problems. This idea led to a new class of spacecraft (SC) flights – low-energy trajectories, in which the change in SC energy during transfer between massive bodies is minimal. One approach to their design is ballistic capture trajectories, which carry out the transit of the SC from one massive body to another. Such trajectories have already been implemented by the Hiten (JAXA), SMART-1 (ESA), Genesis and GRAIL (NASA), Danuri (KARI), etc.

Ballistic capture trajectories in the design of lunar missions began to be studied in the works of V.A. Egorov, V.G. Fesenkov, M.C. Davidson and others. The use of the four-body problem and the WSB (Weak Stability Boundary) trajectory proposed by E.A. Belbruno became important in the design of lunar trajectories.

We call capture the transition of a spacecraft to an orbit with a negative Keplerian energy of a massive body from the outer part of space.

1. The three-body problem

For studying the features of spacecraft motion in a system of two massive bodies, the most useful are the restricted circular and restricted elliptic three-body problems (RC3BP and RE3BP) [1].

The RC3BP analysis shows that transit from one massive body to another with minimal energy change occurs near the L1 and L2 libration points with near-zero velocity [1, 2], which corresponds to the minimal change in the Jacobi constant for the transfer.

Such transit trajectories can be obtained on the basis of stable and unstable invariant manifolds of libration points. Trajectories based on invariant manifolds are in the plane of motion of massive bodies, which determines the planes of satellite orbits that can be obtained from massive bodies without additional maneuvers to change the plane of the orbit. This is a disadvantage for their practical use. The plane (inclination) of the satellite's orbit can be changed by changing the SC's velocity at the libration point.

The capture duration and the suitable orbital parameters are important. The Jacobi integral, Tisserand's parameter and minimum velocity surface are used to study them [1, 3, 4].

In RE3BP, libration points are only a geometric concept and are not a solution in this model. That is, libration points have an instantaneous velocity corresponding to the pulsation of the coordinate system. The region of possible motions (zero velocity curves, Jacobi constant) and transit trajectories depend on the mass parameter of the problem, the eccentricity and the true anomaly of the small massive body [5, 6, 4].

An analysis of the system linearized in the vicinity of the libration point shows that transit at the libration point (with zero velocity in the pulsating coordinate system) is not possible at any moment in time, and depends on the true anomaly of the small body. And for transit, a suitable Jacobi constant is not enough. The correct velocity vector is also necessary [7].

The ranges of true anomaly that allow transit are in the vicinity of $90°$ and $270°$, which correspond to the radial velocity maxima in the orbital coordinate system. In these cases, the transit has a different direction. For example, in the case of the Earth-Moon system for L1 with a true anomaly of $47.64° - 132.90°$, the transit is from the Moon to the Earth, and with $-47.64° - -132.90°$, from the Earth to the Moon [7, 8].

2. The four-body problem

To use these solutions in the perturbed ephemeris model, we will move from a rotating coordinate system associated with the barycenter of the system or the libration point to a stationary coordinate system associated with one of the massive bodies.

It is obvious that in the perturbed four-body model there will be a significant perturbation of the trajectories under consideration. The choice of the date of the libration point flight allows us to determine the transit trajectory formed by invariant manifolds that stays for a sufficiently long time near the small body. For example, for the Earth-Moon system, the duration of stay at the Moon during the flight of the L1 libration point on certain dates is more than 1000 days [7].

Conclusion

A method is proposed for determining the capture orbit near the libration points L1 and L2, based on their invariant manifolds. The analysis of ballistic capture trajectories based on invariant manifolds of libration points is carried out in models of restricted circular, elliptical and perturbed three-body problems. Such transit trajectories were investigated depending on the masses of the bodies and the eccentricity of the orbit in RE3BP. The possibility of determining the trajectories of ballistic capture by selecting the date of the libration point flyby and the velocity vector in the ephemeris model is shown.

Transit trajectories of this type make it possible to obtain ballistic capture orbits suitable for the implementation of low-thrust spacecraft. In particular, such examples in the Earth-Moon system were obtained in [7, 8], and the use of libration points for interplanetary transfers was considered in [9].

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