The ephemeris of the Moon in the framework of the numerical theory of Solar system bodies EPM, IAA RAS

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Abstract. Since 1969, the laser observations of the Moon (LLR) have been used to build support and improve the parameters of the Moon EPM ephemeris within the ERA7 system. The results of processing new LLR observations to obtain refined parameters of the Moon EPM2023a ephemeris within the framework of the modernized ERA8 system are considered. In order to ascertain the parameters of the Moon's ephemeris, 33602 observations of LLR (normal points - n.p., 1985-new) are used. About 100 parameters of the ephemeris of the Moon EPM2023a were improved and compared with some parameters of the ephemerides INPOP21a (France) and DE440 (USA). The values of individual parameters in different ephemerides are generally close. In some cases, discrepancies require a careful review of the list of parameters.

Introduction

The main attention will be paid to the problems of clarifying the parameters of the ephemeris of the Moon EPM2023a. Currently, there are 3 centers where accurate ephemerides of Solar System bodies are created and maintained: DE (USA), INPOP (France), and EPM (Russia). From 1989 till 2014 years, the ephemeris EPM-ERA was developed and supported on the basis of the model of the Moon's motion of Krasinsky G.A. and realized within the ERA-7 system [1]. From 2014, it was being developed a new version of the EPM ephemeris within the modernized ERA-8 system [2]. The geophysical and geodynamical parameters recommended by IERS were included. To obtain and refine the parameters of the Moon's ephemeris, EPM2023a uses 33602 observations of LLR n.p. About 100 parameters of the ephemeris of the Moon EPM2023a were improved and compared with some parameters of the ephemerides INPOP21a and DE440.

Sations	**Years**	**Normal points**	**Added**
Grasse, France (IR)	2015-2023	8479	1632
Matera , Italy	2003-2023	460	13
Apache Point, USA	2006-2023	4126	225
Wettzell, Germany	2018-2023	329	115
Total	**1969-2023**	**33602**	**1985**

TABLE 1. LLR Observations 1969 - 2023

Model of the orbital-rotational motion of the Moon

When constructing the model of the orbital-rotational motion, the Moon is considered an elastic body with a rotating liquid core. The model in EPM2023a is constructed by joint numerical integration of the relativistic Einstein-Infeld-Hoffman equations using the extended Adams method in the inertial BCRS using the TDB dynamic scale, taking into account the compression of the Sun, additional disturbances from the largest asteroids (277), asteroid belts, TNO (30), and the TNO ring. The rotation of the Moon around the center of mass in the celestial coordinate system is given by Euler's three angles, which participate in numerical integration together with the position of the Moon's center. Changes of the model of the orbital-rotational motion of the Moon (in the ERA-8 system) are taken into account in the processing of LLR observations and obtaining new parameters of the ephemeris of the Moon. The i-th body (for example, Moon) in a rectangular and nonrotating coordinate system with the origin at the barycenter of the solar system at the epoch J2000 is as follows:

$$\ddot{\mathbf{r}}_{i_{\text{point mass}}} = \sum_{j\neq i}\frac{\mu_j}{r_{ij}^3}(\mathbf{r}_j-\mathbf{r}_i)\Big\{1-\frac{2(\beta+\gamma)}{c^2}\sum_{k\neq i}\frac{\mu_k}{r_{ik}}-\frac{2\beta-1}{c^2}\sum_{k\neq j}\frac{\mu_k}{r_{jk}}+ \gamma\left(\frac{|\dot{r}_i|}{c}\right)^2+(1+\gamma)\left(\frac{|\dot{r}_j|}{c}\right)^2-\frac{2(1+\gamma)}{c^2}\dot{\mathbf{r}}_i\dot{\mathbf{r}}_j-\frac{3}{2c^2}\left[\frac{(\mathbf{r}_i-\mathbf{r}_j)\times\dot{\mathbf{r}}_j}{r_{ij}}\right]^2+ +\frac{\ddot{\mathbf{r}}_j}{2c^2}(\mathbf{r}_j-\mathbf{r}_i)\Big\}+\frac{1}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}^3}\{[\mathbf{r}_i-\mathbf{r}_j]\times[(2+2\gamma)\dot{\mathbf{r}}_i-(1+2\gamma)\dot{\mathbf{r}}_j]\} (\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_j)+\frac{(3+4\gamma)}{2c^2}\sum_{j\neq i}\frac{\mu_j\ddot{\mathbf{r}}_j}{r_{ij}} \tag{1}$$

It is also necessary to add terms containing the effect of the compression of the Sun:

$$C = 3J_2\mu_s\frac{R^2}{r_{is}^4}\left\{\left[\frac{5}{2}\left(\frac{r_i-r_s}{r_{is}}\times\rho\right)^2-\frac{1}{2}\right]\frac{r_i-r_s}{r_{is}}-\left(\frac{r_i-r_s}{r_{is}}\times\rho\right)\rho\right\} \tag{2}$$

as well as terms containing the Lense-Thirring acceleration:

$$D = \frac{2}{c^2}GS_{Sun}\frac{1}{r_{iS}^3}M_{Sun}\left(\dot{\mathbf{r}}_{iS}\times\mathbf{z}+3\frac{\mathbf{z}\times\mathbf{r}_{iS}}{r^2}\mathbf{r}_{iS}\times\dot{\mathbf{r}}_{iS}\right) \tag{3}$$

Lunar Laser Ranging observations (LLR)

The EPM2023a lunar ephemeris was produced using 33,602 LLR observations between 1969 and 2023. The parameters of the EPM2023a lunar ephemeris were updated with a total of 1985 new LLR observations, which were added to the earlier ones. The number of added observations between 1969 and 2023 is shown in Table 1.

Refl.	X_{DE440}	X_{EPM23}	Y_{DE440}	Y_{EPM23}	Z_{DE440}	Z_{EPM23}
A11	1591967.049	1591966.865	690698.573	690699.163	21004.461	21003.740
A14	1652689.369	1652689.637	-520998.431	-520997.770	-109729.869	-109730.550
A15	1554678.104	1554678.476	98094.498	98095.267	765005.863	765005.253
L1	1114291.452	1114292.318	-781299.273	-781298.603	1076059.049	1076058.727
L2	1339363.598	1339363.663	801870.995	801871.636	756359.260	756358.713

TABLE 2. The coordinates of the reflectors in the ephemerides DE440 and EPM2023a (in meters)

Results of processing new observations

During the processing of LLR observations, the parameters (about 100) at the epoch JD 2446000.5 have been clarified. The new parameters of ephemeris EPM 2023a were compared with some parameters of the ephemerides DE440 and INPOP2021a (see Tables 2 and 3), and the results of processing LLR observations and comparison with parameters of the ephemeris EPM2022 are demonstrated in the Table 4.

Parameter	INPOP 2021a	EPM2023a	INPOP21a – EPM2023a
f_c - core compression ratio	2.8E-04	2.505E-04 ± 0.018E-04	0.295E-04
C_32 - moon potential parameter	4.84501E-06	4.93236E-06 ± 0.00044E-06	-0.08735E-06
h_2 - Moon – Love number	4.23E-02	4.47E-02 ± 0.03E-02	-0.24E-02
k_v/C_T - coefficient of friction between the core the crust	1.62E-08	1.61E-08 ± 0.01E-08	0.01E-08
τ_m - lunar tidal delay (days)	9.6E-02	9.6E-02 ± 0.1E-02	0.00E-02
Rotational delays of the earth tides τ_{R1} (days)	8.02E-03	7.664E-03 ± 0.019E-03	0.356E-03
Rotational delays of the earth tides τ_{R2} (days)	2.82E-03	2.859E-03 ± 0.002E-03	-0.039E-02

TABLE 3. Some parameters of the Earth-Moon system in INPOP21a and EPM2023a

Conclusion

1. The new values the parameters of ephemeris Moon EPM 2023a were obtained during processing of new LLR observations (1985 n.p.) on four 4 stations taking into account all past observations from 1969 till 2023.
2. There are next step for clarify parameters of ephemeris:
   (a) It is necessary to continue to study cases of deviation of erroneous observations at various stations and introduce biases.
   (b) In the papers [3, 4] based on mathematical modeling, it was shown that there are some ways to improve the parameters of lunar ephemeris: adding new observation stations (up to 12%); involvement of radar observations of the Moon (from 20% to 60%); as well as VLBI observations (at the LLR accuracy level).

Station	EPM 2022a				EPM 2023a			
	Years	**Normal points num.**	**Disc.**	**rms (cm)**	**Years**	**Normal points num.**	**Disc.**	**rms (cm)**
McDonald	1970-1985	3588	34	21.3	1970-1985	3588	34	21.7
MLRS1	1983-1988	631	46	8.8	1983-1988	631	46	8.8
MLRS2	1988-2015	3669	388	3.5	1988-2015	3669	388	3.6
Haleakala	1984-1990	770	22	5.1	1984-1990	770	22	5.3
Cerga Ruby	1984-1986	1112	3	16.7	1984-1986	1112	3	16.7
Cerga YAG	1987-2005	8316	39	2.3	1987-2005	8316	40	2.3
Cerga MeO	2009-2022	2097	0	1.5	2009-2022	2097	0	1.5
Cerga IR	2015-2022	6847	7	1.2	2015-2023	8479	4	1.4
Apache	2006-2022	3901	78	1.4	2006-2023	4126	78	1.5
Matera	2003-2022	421	2	3.2	2003-2023	460	28	3.4
Wettzell	2018-2022	212	0	1.5	2018-2023	329	3	1.6

TABLE 4. The comparison parameters of the ephemerides EPM2023a and EPM2022a

(c) Regarding the use of lunar radar observations (LRR): there are already real observations [5], which were used to obtain selenocentric coordinates of the lander and other parameters (joint work with Chinese colleagues) - observations at the level of 1-3 mm [6].

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