Numerical-analytical approach to the study of resonant structures of nearplanetary orbital spaces

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Abstract. A numerical and analytical method for studying the resonant structures of nearplanetary spaces and the results of its application to the construction of such structures for the Earth and the Moon are presented.

Introduction

The idea of joint use of analytical and numerical approaches in the analysis of resonances in dynamic systems was first expressed by B.V. Chirikov [1]. And in problems of celestial mechanics, this idea was first applied in works [2, 3] devoted to the dynamics of objects in GPS systems and the developed GALILEO system.

1. Research methodology

Numerical modeling is used to calculate the orbital evolution of objects over a selected time interval. The software package “Numerical model of the motion of satellite systems” is used. The latest version of software package is described in [4]. The software package is implemented in a parallel computing environment on the supercomputer "SKIF Cyberia" of Tomsk State University.

Numerical modeling allows us to obtain an array of position vectors and osculating orbital elements of all objects under consideration at given moments in time For the same moments in time, the values of the fast Lyapunov characteristic MEGNO. The components of the frequency basis are determined using numerical and analytical approaches [5].

Formulas for searching for resonance characteristics are found using analytical methods. Formulas for searching for resonance characteristics are found using analytical methods.

Resonance (critical) arguments and resonance relations for orbital (tesseral) resonances are formed using the technique proposed by R. Alan [6, 7], refined by E. D. Kuznetsov [8] for resonance 1:3 and generalized in [9].

To obtain the characteristics of secular and semi- secular resonances, the technique proposed by J. Cook [10] is used. These characteristics are extracted from the argument of the once and twice averaged perturbing function. In our work, we considered the following types of secular (table 1 in [9]) and semi-secular (table 2 in [9]) resonances.

Secular frequencies in satellite motion are calculated both by numerical modeling [11] and by well-known analytical formulas.

2. Results

Using the methodology described above, extensive numerical and analytical experiments were carried out to analyze the resonant structures of near-Earth space (NES) and near-Lunar space (NLS).

The dynamics of NES objects was analyzed in the range of semimajor axes from 8000 to 315000 km with a step of 200 km and inclinations from 0 to 180° with a step of 5°, and an initial eccentricity equal to 0.001.

In this case, disturbances from harmonics of the geopotential up to 10th order and degree were taken into account, as well as disturbances from the Moon and the Sun. Together with the equations of motion, the equations of the parameters of MEGNO, designed to identify chaos in the dynamics of objects, were integrated.

The dynamics of objects in the regions of orbital resonances 1:1 – 1:11 in the direction of decreasing semi-major axis of the satellite's orbit, as well as 2:1 and 3:1 in the direction of its increase, are examined in detail.

A comparison of the features of the evolution of objects in the non-resonance zone with the evolution of objects moving in the orbital resonance zones showed that the movement in the resonant zones is more chaotic.

If in a non-resonant zone the phenomenon of chaotic movement is rarely observed, then for resonant zones it is a characteristic property. The determining factor in the occurrence of chaos in the movement of objects is the presence in the dynamics of unstable components of the orbital resonance.

The influence of orbital resonance on the occurrence of chaoticity is so great that chaoticity manifests itself even in cases where all components of orbital resonance have circulating resonant arguments, but the resonance ratios repeatedly pass through zero values and all other sources of chaotic occurrence are absent.

The contribution of secular resonances to the emergence of chaos in resonant zones is secondary, but they have an impact on the orbital evolution, which is manifested by an increase in the amplitudes of long-period oscillations of positional variables.

This applies, first of all, to secular resonances of the first order: apsidal geometric resonance of the Lidov–Kozai type $\underline{\underline{\dot{\psi}}} = \dot{\omega} \approx 0$ and nodal resonance $\underline{\underline{\dot{\psi}}} = (\dot{\Omega} - \dot{\Omega}'_{S,L}) \approx 0$.

It should be noted that the zone in which the influence of the Lidov-Kozai type resonance is manifested extends along the semi-major axis from 20,000 km to 260,000 km in the region of forward motion and up to 100,000 km in the region of reverse motion. And in terms of inclination, the zone occupies an area from 60 to 120° in the interval of semi-major axes from 20,000 km to 100,000 km; above these values, it is present mainly in the zone of direct movement and is concentrated around an inclination of 90°.

As for the nodal resonance, it appears near inclinations of 0, 90 and 180°, and near inclinations of 0 and 180° – in the range of semi-major axes from 10,000 km to 250,000 km, and in the vicinity of 90° from 10,000 to 110,000 km.

Resonances with the average motion of the third body are present only in low orbits and their influence is insignificant.

It is interesting to note that in the dynamics of objects, the superposition of several stable secular resonances does not lead to the appearance of chaos, and on the contrary, the combined action of stable and unstable secular resonances causes the appearance of chaos in the movement of an object.

To study the dynamic structure of the near-Lunar orbital space using the software "Numerical model of the motion of artificial lunar satellites", the motion of 5180 objects was simulated over a 10-year time interval. The initial position of each satellite was characterized by a circular orbit and its own values of semi-major axis and inclination. The elements a and i were varied in incremental and 5-degree incremental ranges $a \in [1.1.R_L; 15R_L]$ with a step of $0.1R_L$ and $i \in [0; 180°]$ with a step of 5°.

The following results were obtained: the short lifetime of objects in low orbits is explained by the direct influence of the complex gravitational field of the Moon; there are no orbital resonances in the motion of the moon's satellites, and semi-secular resonances are still unstable. Thus, the main resonant factor in the motion of the lunar satellites are secular apsidal-nodal resonances, and the Lidov–Kozai type resonance and low-order nodal resonances have the greatest influence.

The Lidov–Kozai type resonance extends in a wide band across the entire considered region of cislunar space in the inclination range from 55 to 110°. Nodal resonance also runs through the entire region and clusters around $i = 90°$.

As was shown in [12] for the 1:1 orbital resonance, the influence of light pressure leads to the appearance of secondary orbital resonances, the areas of action of which above and below along the semimajor axis cover the area of action of the main resonance. Our numerical and analytical modeling allows us to state that all components of all considered orbital resonances have secondary analogues, which leads to a significant expansion of the bands of orbital resonances.

Conclusion

Thus, the numerical-analytical technique makes it possible to obtain a large number of interesting and useful results in the study of resonant structures of near-planetary orbital spaces.

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